

On the Range Spectral Loss in InSAR

Yuhang Wang, Cunren Liang, Xue Li, Fan Yang, Kunyi Chen

Abstract—Range spectral loss, a common issue in Synthetic Aperture Radar Interferometry (InSAR) caused by the wavenumber shift, Radio Frequency Interference (RFI), and intentional bandpass filtering, has not been systematically characterized, potentially leading to misinterpretations of its effects. To address this problem, we develop a comprehensive, end-to-end analytical framework based on the spectral representation of the SAR range signal. Within this framework, we systematically derive the effects of range spectral loss on the point target response, the interferogram, and the estimates obtained via the range split-spectrum method. We find that non-dispersive processes do not affect these outputs; instead, effects arise solely from dispersive processes, primarily that associated with the ionosphere. The impacts on interferograms are generally negligible. However, the errors introduced into quantities estimated by the range split-spectrum method can be nonnegligible, depending on the observation conditions. These findings clarify potential misinterpretations, alleviate concerns about interferograms, and draw attention to the possible effects on quantities derived from the range split-spectrum method in the presence of range spectral loss.

Index Terms—Radio Frequency Interference (RFI), range split-spectrum method, spectrum, Synthetic Aperture Radar Interferometry (InSAR), wavenumber shift

NOMENCLATURE

DIP	Dominant Ionospheric Phase.
SIP	Secondary Ionospheric Phase.
CSIP	Constant Secondary Ionospheric Phase.
VSIP	Varying Secondary Ionospheric Phase.
PPV	Peak Phase Variation.
OPV	Original-peak-position Phase Variation.
SSP	Spectral-Shifting Phase.
SSIP	Spectral-Shifting Ionospheric Phase.
DDIP	Differential Dominant Ionospheric Phase.
DCSIP	Differential Constant Secondary Ionospheric Phase.
DSSIP	Differential Spectral-Shifting Ionospheric Phase.
PVP	Position Variation Phase.
BVP	Bandwidth Variation Phase.
SAR	Synthetic Aperture Radar.
InSAR	Synthetic Aperture Radar Interferometry.
RFI	Radio Frequency Interference.
TEC	Total Electron Content.

FBD	Fine mode Dual polarization (ALOS-2/4).
FWD	Fine mode Wide Dual polarization (ALOS-4).
HBQ	High-sensitive mode Full (Quad.) polarimetry (ALOS-2/4).
UBS	Ultra-fine mode Single polarization (ALOS-2/4).
UWD	Ultra-fine mode Wide Dual polarization (ALOS-4).
WBD	ScanSAR nominal [14MHz] mode Dual polarization (ALOS-2).

I. INTRODUCTION

TRANSMITTING a wide-bandwidth pulse is fundamental to achieving high range resolution in Synthetic Aperture Radar (SAR). However, the wide-bandwidth range spectrum is frequently subject to spectral loss, particularly when the data are used in SAR interferometry (InSAR). The most prevalent form of the range spectral loss arises from the range spectral shift induced by the perpendicular baseline [1]:

$$2\Delta f = \frac{f_c B_\perp}{R \tan(\theta - \alpha)} \quad (1)$$

where f_c is the radar carrier frequency, $B_\perp$ is the perpendicular baseline, R is the slant range, θ is the look angle, and α is the terrain slope angle. The factor of 2 before Δf is introduced here to facilitate subsequent analysis. In view of this problem, the orbits of modern SAR satellites are tightly controlled to minimize the range spectral shift, often leading to an orbital tube diameter of less than 500 m [2-5]. Consequently, the resulting $2\Delta f$ is generally considered small. However, it also depends on the look angle and terrain slope angle, which can render it appreciable or even significant. Specifically, substantial variation in $2\Delta f$ may occur due to the large look angle variations from near to far range in modern satellite SAR systems, which are often characterized by a wide swath (e.g., >200 km). Furthermore, an even more pronounced $2\Delta f$ can be caused by the terrain slope. For a slope facing toward the radar, where the slope angle approaches the look angle, even a minimal perpendicular baseline can induce an exceptionally large $2\Delta f$. In contrast, for slopes facing away from the radar, $2\Delta f$ remains minimal.

The Radio Frequency Interference (RFI) represents another significant source of range spectral loss. The RFI refers to undesired signals from other radiation sources operating in the same frequency band that are captured by the SAR system. It has become increasingly prevalent due to the growing congestion of electromagnetic spectrum resources, particularly within the long-wavelength bands [6]. Such interference can severely degrade SAR data quality, consequently undermining a wide range of downstream applications [7]. A variety of RFI mitigation techniques have been developed [8]. The simplest

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and most widely adopted approach is notch filtering in the range-frequency domain, which inevitably leads to range spectral loss. Such an approach is routinely adopted in a number of SAR missions, including the ALOS series [9], [10], Sentinel-1 [11], and LuTan-1 [12], as well as upcoming missions such as ROSE-L [13].

Range spectral loss can also arise from intentional bandpass filtering of the range spectrum. For example, the range split-spectrum method employs bandpass filtering to generate two or more sub-bands. This method has been used to determine the absolute InSAR phase [14], estimate the residual range offset between a pair of images (i.e., the spectral-diversity method) [15], and estimate the InSAR ionospheric phase [16]. Notably, ionospheric phase estimation via the range split-spectrum method has become an indispensable component of modern L-band SAR systems and InSAR software packages. The total range spectral loss incurred by applying this method thus stems from a combination of bandpass filtering, range spectral shift, and/or the RFI, thereby resulting in a highly complex scenario.

Range spectral loss is inherently linked to the radar carrier frequency (or wavelength), a fundamental parameter that governs phase measurements in InSAR. Moreover, the frequency-independent (non-dispersive) and frequency-dependent (dispersive) components of the InSAR signal are affected differently in the presence of range spectral loss. A critical question therefore arises: Does range spectral loss introduce significant biases into the *expected* InSAR phase? Furthermore, does it introduce significant biases into the various quantities estimated by the range split-spectrum method? And if so, can these biases be rigorously quantified? Despite the prevalence of range spectral loss in InSAR, these problems remain insufficiently investigated and are not yet fully understood. For example, confusion associated with range spectral loss has arisen in the range split-spectrum method for estimating the InSAR ionospheric phase [16] and in the calculation of the effective vertical interferometric wavenumber used in PolInSAR [17]. Moreover, determining whether range spectral loss contributes to the range ramp frequently observed in large-scale InSAR measurements is of great interest, given the appreciable variations in range spectral loss from near to far range induced by the range spectral shift in wide swath modes.

In this work, we conduct a systematic and comprehensive investigation of the range spectral loss in InSAR through an end-to-end analysis based on the spectral representation of the SAR signal. The remainder of the paper is organized as follows. Section II analyzes the effects of range spectral loss on the point target response. Sections III and IV examine its effects on InSAR and on the range split-spectrum method, respectively. In Sections V and VI, the key theoretical findings are validated by both simulations and real-data experiments. Section VII provides further discussion on related issues. Finally, Section VIII concludes the paper.

II. POINT TARGET RESPONSE

A. Point Target Response in the Absence of the Ionosphere

Based on (A.8) in Appendix A, the spectrum of the point target response can be expressed as

$$S_{\text{focus}}(f) = \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} \cdot H_{\text{bs}}(f + f_c) \cdot H_{\text{geom}}(f + f_c) \cdot H_{\text{tropo}}(f + f_c) \cdot H_{\text{iono}}(f + f_c) \quad (2)$$

where f is the range frequency, f_c is the carrier frequency, K_r is the linear FM rate, T_r is the signal duration, and $H_{\text{bs}}(f)$, $H_{\text{geom}}(f)$, $H_{\text{tropo}}(f)$, and $H_{\text{iono}}(f)$ are the transfer functions of the target backscattering, geometric, tropospheric, and ionospheric systems, respectively. For each physical process, we assume that different frequency components experience uniform attenuation within the spectral range of the SAR system; therefore, attenuation can be neglected without affecting the subsequent analysis. Within the spectral range of the SAR system, the target reflectivity can be assumed to be invariant:

$$H_{\text{bs}}(f) = \exp\{j\varphi_{\text{bs}}\} \quad (3)$$

where φ_{bs} is a constant phase. The geometric effect is caused by a constant time delay t_{geom} :

$$H_{\text{geom}}(f) = \exp\{-j2\pi f t_{\text{geom}}\} \quad (4)$$

The troposphere is generally considered a non-dispersive medium [18], although some studies indicate that heavy rain can induce weak dispersive effects [19]. Its effect can be represented by a constant time delay t_{tropo} :

$$H_{\text{tropo}}(f) = \exp\{-j2\pi f t_{\text{tropo}}\} \quad (5)$$

Therefore, all these physical processes can be classified as non-dispersive processes that are frequency-independent. In contrast, the ionospheric effect is dispersive, which will be addressed in Section II-B.

The time-domain point target response is obtained by applying the inverse Fourier transform to the spectrum:

$$s_{\text{focus}}(t) = \int_{-\infty}^{\infty} S_{\text{focus}}(f) \exp\{j2\pi f t\} df \quad (6)$$

Its phase comprises three components:

- (1) a constant phase, $\varphi_{\text{non-disp}}$, arising from non-dispersive processes, including backscattering, geometric delay, and tropospheric delay;
- (2) a constant phase, φ_{iono} , induced by the dispersive ionosphere; and
- (3) a time-varying phase, $\varphi_{\text{tropo}}(\Delta t)$, which characterizes the phase variation with respect to the peak position, where Δt denotes the time offset relative to the peak position.

Considering the non-dispersive processes alone, $H_{\text{bs}}(f + f_c)$, $H_{\text{geom}}(f + f_c)$, and $H_{\text{tropo}}(f + f_c)$ contain only constant and linear terms with respect to frequency. After the inverse Fourier transform, the constant term manifests as a constant phase in the time domain, while the linear term leads to a shift in the target position. Consequently, (6) can be expressed as

$$\begin{aligned} s_{\text{focus}}(t) &= T_r \text{sinc}\{K_r T_r (t - t_{\text{geom}} - t_{\text{tropo}})\} \\ &\quad \cdot \exp\{j(\varphi_{\text{bs}} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}})\} \\ &= T_r \text{sinc}\{K_r T_r (t - t_{\text{geom}} - t_{\text{tropo}})\} \\ &\quad \cdot \exp\{j\varphi_{\text{non-disp}}\} \end{aligned} \quad (7)$$

where

$$\varphi_{\text{non-disp}} = \varphi_{\text{bs}} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}} \quad (8)$$

To summarize, these non-dispersive processes lead only to a frequency-independent delay in the target position and a constant phase that depends on the radar carrier frequency. When expressed in the same units, these two quantities are equal if the backscattering phase is excluded. No time-varying phase is generated.

B. Point Target Response in the Presence of the Ionosphere

Neglecting the bending of the propagation path, the two-way ionospheric phase for a monochromatic wave can be expressed as [20]

$$\begin{aligned} \phi_{\text{iono}}(f) &= \frac{4\pi f}{c} \left[-\frac{K_2}{f^2} \int N dl_0 \pm \frac{K_3}{f^3} \int N B_{\text{mag}} \cos \theta dl_0 \right. \\ &\quad \left. - \frac{K_{4,1}}{f^4} \int N^2 dl_0 \right. \\ &\quad \left. - \frac{K_{4,2}}{f^4} \int N B_{\text{mag}}^2 (1 + \cos^2 \theta) dl_0 \right] \\ &\approx -\frac{4\pi K_2}{cf} \int N dl_0 \\ &= -\frac{4\pi K_2}{cf} \text{TEC} \\ &= -\frac{4\pi K_2}{cf_c} \text{TEC} \sum_{n=0}^{+\infty} \left(-\frac{f-f_c}{f_c} \right)^n \\ &\approx -\frac{4\pi K_2}{cf_c} \text{TEC} \left[1 - \frac{f-f_c}{f_c} + \left(\frac{f-f_c}{f_c} \right)^2 \right] \end{aligned} \quad (9)$$

where N is the electron number density, B_{mag} is the Earth's magnetic flux density, θ is the angle between the Earth's magnetic field and the electromagnetic wave propagation direction, TEC is the total electron content along the propagation path, and c is the speed of light in vacuum. The constants are defined as $K_2 = 40.31 \text{ m}^3/\text{s}^2$, $K_3 = 1.13 \times 10^{12} \text{ A} \cdot \text{m}^3/(\text{s} \cdot \text{kg})$, $K_{4,1} = 812.38 \text{ m}^6/\text{s}^4$, and $K_{4,2} = 1.58 \times 10^{22} \text{ A}^2 \cdot \text{m}^3/\text{kg}^2$. Two approximations are introduced in (9). The first retains only the leading term in the square brackets, while the second retains terms up to the second order in the Taylor series expansion. The errors introduced by these approximations are sufficiently small for the following analysis, as demonstrated in Appendix B.

Note that the negative sign of $\phi_{\text{iono}}(f)$ in (9) indicates that it represents a phase advance under the adopted convention. The transfer function of the ionospheric system can be expressed as [21], [22]

$$\begin{aligned} H_{\text{iono}}(f + f_c) &= \exp\{-j\phi_{\text{iono}}(f + f_c)\} \\ &= \exp\left\{j \frac{4\pi K_2}{cf_c} \text{TEC} \left[1 - \frac{f-f_c}{f_c} + \left(\frac{f-f_c}{f_c} \right)^2 \right] \right\} \\ &= \exp\{j(\mu_0 - 2\pi\mu_1 f + \pi\mu_2 f^2)\} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \mu_0 &= \frac{4\pi K_2}{cf_c} \text{TEC} \\ \mu_1 &= \frac{2K_2}{cf_c^2} \text{TEC} \\ \mu_2 &= \frac{4K_2}{cf_c^3} \text{TEC} \end{aligned} \quad (11)$$

The three terms in the last line of (10) represent distinct ionospheric effects:

(1) μ_0 is a constant term, corresponding to a two-way constant phase advance in the time domain, termed the Dominant Ionospheric Phase (DIP), φ_{DIP} .

(2) $-2\pi\mu_1 f$ is a linear term, corresponding to a two-way position delay of μ_1 in the time domain. The position delay and the phase advance are equal in magnitude when expressed in the same units.

(3) $\pi\mu_2 f^2$ is a quadratic term that alters the time-frequency relationship of the linear FM signal, leading to an FM rate mismatch of $\Delta K = \mu_2 K_r^2$ during focusing. The inverse Fourier transform of this phase term is derived in Appendix C.

By incorporating $H_{\text{iono}}(f + f_c)$, the time-domain point target response can be expressed as

$$s_{\text{focus}}(t) = T_r \exp\{j(\varphi_{\text{bs}} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}})\} \cdot \exp\{j\mu_0\} s_{\text{qdt}}(t - t_{\text{geom}} - t_{\text{tropo}} - \mu_1) \quad (12)$$

where $s_{\text{qdt}}(t)$ is defined in (C.1) and approximated analytically in (C.5). The peak position is located at

$$\begin{aligned} t_{\text{peak}} &= t_{\text{geom}} + t_{\text{tropo}} + \mu_1 \\ &= t_{\text{geom}} + t_{\text{tropo}} + \frac{2K_2}{cf_c^2} \text{TEC} \end{aligned} \quad (13)$$

As shown in Appendix C, the quadratic phase term introduced by the ionosphere gives rise to a time-domain phase, termed the Secondary Ionospheric Phase (SIP), which can be expressed as the sum of a Constant Secondary Ionospheric Phase (CSIP), φ_{CSIP} , and a time-Varying Secondary Ionospheric Phase (VSIP), φ_{VSIP} , with the time measured relative to the peak position. Therefore, the phase of the point target response can be expressed as

$$\begin{aligned} \varphi_{\text{focus}} &= \varphi_{\text{non-disp}} + \varphi_{\text{iono}} \\ &= \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} + \varphi_{\text{SIP}} \\ &= \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} + \varphi_{\text{CSIP}} + \varphi_{\text{VSIP}} \end{aligned} \quad (14)$$

Within a region centered around the peak position with a radius of $1/\pi$ resolution cell, termed the phase-stable region, the VSIP can be neglected. The CSIP can be calculated as

$$\varphi_{\text{CSIP}} = \frac{\text{QPE}}{3} = \frac{1}{3} \Delta K \left(\frac{T_r}{2} \right)^2 = \frac{\pi B^2}{12} \mu_2 = \frac{\pi K_2 B^2}{3cf_c^3} \text{TEC} \quad (15)$$

where the QPE is the Quadratic Phase Error, and $B = |K_r|T_r$ is the signal bandwidth. The phase at the peak position is directly obtained by removing the VSIP from (14):

$$\begin{aligned} \varphi_{\text{peak}} &= \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} + \varphi_{\text{CSIP}} \\ &= \varphi_{\text{non-disp}} + \mu_0 + \frac{\text{QPE}}{3} \\ &= \varphi_{\text{non-disp}} + \mu_0 + \frac{\pi B^2}{12} \mu_2 \\ &= \varphi_{\text{bs}} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}} + \frac{4\pi K_2}{cf_c} \text{TEC} \\ &\quad + \frac{\pi K_2 B^2}{3cf_c^3} \text{TEC} \end{aligned} \quad (16)$$

The CSIP is significantly smaller than the DIP. Assuming a TEC value of 60 TECU, representing a high-TEC level for typical SAR acquisitions, the ALOS-4 UWD mode exhibits the largest CSIP among all cases listed in Table B.I. Nevertheless, the resulting CSIP is only 0.29 rad, whereas the corresponding DIP reaches 806 rad.

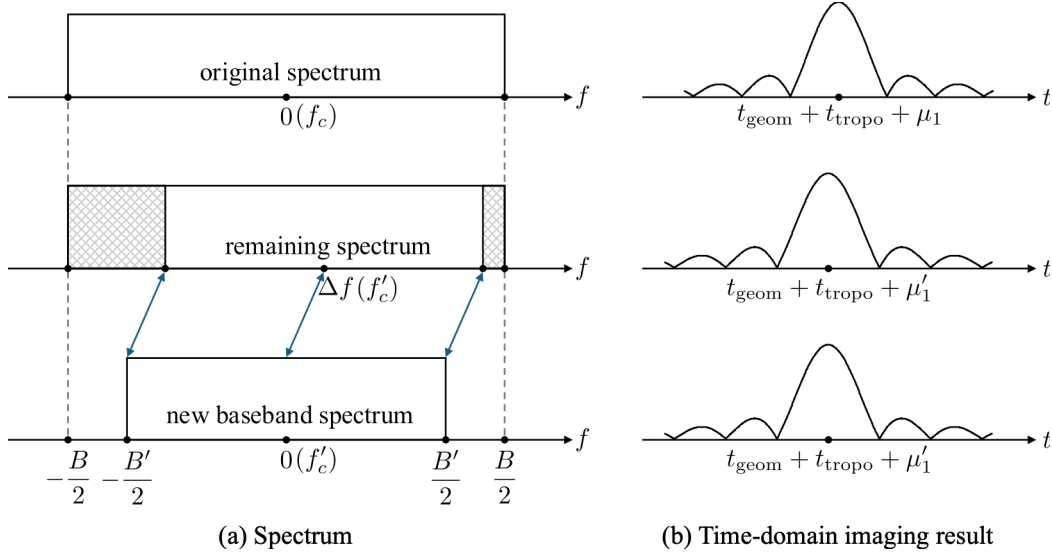


Fig. 1. Illustration of the range spectral loss occurring exclusively at the ends of the spectrum.

C. Effects of Range Spectral Loss in a Simplified Scenario

Spectral loss can occur at any position within the range spectrum. We begin with a scenario where spectral loss occurs exclusively at the ends of the range spectrum, i.e., the remaining spectrum consists of a single continuous segment, as shown in Fig. 1. The center frequency of the remaining spectrum is then shifted from 0 to Δf . To derive the resulting point target response, we first assume a hypothetical SAR system with a carrier frequency of $f'_c = f_c + \Delta f$ and a bandwidth of B' , corresponding to the new baseband spectrum shown in Fig. 1.

Following (13) and (16), the peak position of the new point target response and the phase at this position under the new SAR system are

$$\begin{aligned} t'_{\text{peak}} &= t_{\text{geom}} + t_{\text{tropo}} + \mu'_1 \\ &= t_{\text{geom}} + t_{\text{tropo}} + \frac{2K_2}{c(f_c + \Delta f)^2} \text{TEC} \end{aligned} \quad (17)$$

and

$$\begin{aligned} \varphi'_{\text{peak}} &= \varphi'_{\text{non-disp}} + \mu'_0 + \frac{\text{QPE}'}{3} \\ &= \varphi_{\text{bs}} - 2\pi(f_c + \Delta f)(t_{\text{geom}} + t_{\text{tropo}}) \\ &\quad + \frac{4\pi K_2}{c(f_c + \Delta f)} \text{TEC} + \frac{\pi K_2 B'^2}{3c(f_c + \Delta f)^3} \text{TEC} \end{aligned} \quad (18)$$

where the superscript ' denotes a change in a parameter associated with the new SAR system. The peak position variation is given by

$$\Delta t_{\text{peak}} = t'_{\text{peak}} - t_{\text{peak}} = \mu'_1 - \mu_1 \approx -\frac{4K_2 \Delta f}{cf_c^3} \text{TEC} \quad (19)$$

Then,

$$|B' \Delta t_{\text{peak}}| = \frac{4K_2 \text{TEC}}{cf_c^3} |B' \Delta f| \leq \frac{K_2 B^2}{2cf_c^3} \text{TEC} \quad (20)$$

According to Appendix C, provided that $|B' \Delta t_{\text{peak}}| \leq 1/\pi$, the original peak position t_{peak} falls within the phase-stable region of the new point target response. Therefore, from (20), a critical TEC can be defined:

$$\text{TEC}_{\text{critical}} = \frac{2cf_c^3}{\pi K_2 B^2} \quad (21)$$

below which the phase of the new point target response at the original peak position can always be approximated by φ'_{peak} . Table I lists the critical TEC values for representative satellites listed in Table B.I, considering only the widest-bandwidth imaging mode for each satellite. The L-band values are significantly lower than those for other bands, yet still exceed 140 TECU. Such high TEC levels are rarely observed in practice; therefore, the above approximation is well justified.

TABLE I
CRITICAL TEC VALUES FOR REPRESENTATIVE SATELLITES.

Satellite	Frequency Band	Critical TEC [TECU]
BIOMASS	P	1083
ALOS-2	L	150
ALOS-4	L	140
LuTan-1	L	141
NISAR	L	159
	S	2758
Sentinel-1	C	23420

By shifting the center frequency of the new baseband spectrum from 0 to Δf , the point target response in the presence of spectral loss under the original SAR system is obtained. The spectral shift does not alter the target position, but introduces a linear phase, $2\pi\Delta f t$, in the time domain. Consequently, the phase at the new peak position becomes

$$\begin{aligned} \varphi_{\text{peak,new}} &= \varphi'_{\text{peak}} + 2\pi\Delta f t'_{\text{peak}} \\ &= \varphi'_{\text{non-disp}} + \mu'_0 + \frac{\text{QPE}'}{3} \\ &\quad + 2\pi\Delta f (t_{\text{geom}} + t_{\text{tropo}} + \mu'_1) \\ &= \varphi_{\text{non-disp}} + \mu'_0 + 2\pi\Delta f \mu'_1 + \frac{\text{QPE}'}{3} \\ &= \varphi_{\text{non-disp}} + \mu_0 + (\mu'_0 - \mu_0 + 2\pi\Delta f \mu'_1) + \frac{\text{QPE}'}{3} \end{aligned} \quad (22)$$

The non-dispersive phase at the new peak position remains unchanged compared with (16). In contrast, the ionospheric phase is altered.

The original peak position lies within the phase-stable region centered at the new peak position. However, the non-zero center frequency introduces a time-varying phase term, $2\pi\Delta f\Delta t$. Therefore, the phase at this position is

$$\begin{aligned}\varphi_{\text{peak,ori}} &= \varphi_{\text{peak,new}} + 2\pi\Delta f(t_{\text{peak}} - t'_{\text{peak}}) \\ &= \varphi_{\text{peak,new}} + 2\pi\Delta f(\mu_1 - \mu'_1) \\ &= \varphi_{\text{non-disp}} + \mu'_0 + 2\pi\Delta f\mu_1 + \frac{\text{QPE}'}{3} \\ &= \varphi_{\text{non-disp}} + \mu_0 + (\mu'_0 - \mu_0 + 2\pi\Delta f\mu_1) + \frac{\text{QPE}'}{3}\end{aligned}\quad (23)$$

We define the difference between the peak phase after spectral loss and that of the original full spectrum as the Peak Phase Variation (PPV):

$$\begin{aligned}\Delta\varphi_{\text{PPV}} &= \varphi_{\text{peak,new}} - \varphi_{\text{peak}} \\ &= (\mu'_0 - \mu_0 + 2\pi\Delta f\mu'_1) + \left(\frac{\text{QPE}' - \text{QPE}}{3}\right) \\ &= \varphi_{\text{SSP}_1} + \Delta\varphi_{\text{CSIP}}\end{aligned}\quad (24)$$

The first component of PPV arises from the variation in the center frequency and, therefore, is termed the Spectral-Shifting Phase (SSP):

$$\begin{aligned}\varphi_{\text{SSP}_1} &= \mu'_0 - \mu_0 + 2\pi\Delta f\mu'_1 \\ &= \frac{4\pi K_2}{c(f_c + \Delta f)}\text{TEC} - \frac{4\pi K_2}{cf_c}\text{TEC} + \frac{4\pi K_2\Delta f}{c(f_c + \Delta f)^2}\text{TEC} \\ &\approx -\frac{4\pi K_2\text{TEC}}{cf_c^3}(\Delta f)^2 \\ &= 2\pi\Delta f\frac{\Delta t_{\text{peak}}}{2}\end{aligned}\quad (25)$$

The approximation in the third line is derived using a Taylor series expansion with respect to Δf up to the second order.

The second component of PPV primarily arises from the variation in the bandwidth:

$$\begin{aligned}\Delta\varphi_{\text{CSIP}} &= \frac{\text{QPE}' - \text{QPE}}{3} \\ &= \frac{\pi K_2 B'^2}{3c(f_c + \Delta f)^3}\text{TEC} - \frac{\pi K_2 B^2}{3cf_c^3}\text{TEC} \\ &\approx \frac{\pi K_2\text{TEC}}{3cf_c^3}(B'^2 - B^2) \\ &= \frac{\pi\mu_2}{12}(B'^2 - B^2)\end{aligned}\quad (26)$$

The approximation in the third line is valid under the condition that $f_c \gg B, B', |\Delta f|$. In the specific case where spectral loss occurs at only one end of the spectrum, we have $B' = B - 2|\Delta f|$. In this case, the approximation in (26) is equivalent to retaining terms up to second order in a Taylor expansion with respect to Δf .

A second phase difference is defined between the new and original point target responses at the original peak position, termed the Original-peak-position Phase Variation (OPV):

$$\begin{aligned}\Delta\varphi_{\text{OPV}} &= \varphi_{\text{peak,ori}} - \varphi_{\text{peak}} \\ &= (\mu'_0 - \mu_0 + 2\pi\Delta f\mu_1) + \left(\frac{\text{QPE}' - \text{QPE}}{3}\right) \\ &= \varphi_{\text{SSP}_2} + \Delta\varphi_{\text{CSIP}}\end{aligned}\quad (27)$$

The corresponding SSP component is given by

$$\begin{aligned}\varphi_{\text{SSP}_2} &= \mu'_0 - \mu_0 + 2\pi\Delta f\mu_1 \\ &= \frac{4\pi K_2}{c(f_c + \Delta f)}\text{TEC} - \frac{4\pi K_2}{cf_c}\text{TEC} + \frac{4\pi K_2\Delta f}{cf_c^2}\text{TEC} \\ &\approx \frac{4\pi K_2\text{TEC}}{cf_c^3}(\Delta f)^2 \\ &= -2\pi\Delta f\frac{\Delta t_{\text{peak}}}{2}\end{aligned}\quad (28)$$

Appendix D elaborates on the underlying mechanism of the SSP. It results from the superposition of two effects induced by the center-frequency shift: one is the variation in the phase of the system transfer function with respect to the center frequency, and the other is the linear phase introduced by shifting the center frequency from 0 to Δf . As a result of the superposition of these two effects,

$$\varphi_{\text{SSP}} = 2\pi\Delta f\Delta t \quad (29)$$

where Δt denotes the time offset relative to the phase-invariant point. This phase-invariant point coincides approximately with the midpoint between the peak positions before and after range spectral loss. Hence, for the PPV and OPV, $\varphi_{\text{SSP}} = \pm\pi\Delta f\Delta t_{\text{peak}}$, which is consistent with (25) and (28). The emergence of the SSP is independent of whether the system is dispersive or non-dispersive. However, the differences among the original peak position, the peak position after range spectral loss, and the phase-invariant point exist solely in dispersive systems. To highlight the influence of the ionosphere, we define the SSP in the OPV as the Spectral-Shifting Ionospheric Phase (SSIP):

$$\varphi_{\text{SSIP}} = \varphi_{\text{SSP}_2} = -\pi\Delta f\Delta t_{\text{peak}} = \frac{4\pi K_2\text{TEC}}{cf_c^3}(\Delta f)^2 \quad (30)$$

Then, $\Delta\varphi_{\text{PPV}} = -\varphi_{\text{SSIP}} + \Delta\varphi_{\text{CSIP}}$, $\Delta\varphi_{\text{OPV}} = \varphi_{\text{SSIP}} + \Delta\varphi_{\text{CSIP}}$.

Using the above definitions, we can reformulate the phase after range spectral loss in (22) and (23) as

$$\begin{aligned}\varphi_{\text{peak,new}} &= \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} - \varphi_{\text{SSIP}} + \varphi'_{\text{CSIP}} \\ \varphi_{\text{peak,ori}} &= \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} + \varphi_{\text{SSIP}} + \varphi'_{\text{CSIP}}\end{aligned}\quad (31)$$

where φ'_{CSIP} denotes the CSIP after range spectral loss. Note that, according to (26), when evaluating φ'_{CSIP} , the variation in center frequency can be neglected, and only the variation in bandwidth needs to be considered.

Since $B \ll f_c$ and $\Delta f \ll f_c$, range spectral loss generally has a negligible effect on the peak phase. Moreover, given that $B' \leq B - 2|\Delta f|$, $\Delta\varphi_{\text{CSIP}}$ is generally larger than φ_{SSIP} when $B > 4|\Delta f|$. For example, if $f_c = 1.258$ GHz, $B = 79.4$ MHz (the parameters of the ALOS-2 UBS mode in Table B.I), $\text{TEC} = 60$ TECU, $\Delta f = 10$ MHz and $B' = B - 2\Delta f$ (corresponding to a range spectral loss of 20 MHz occurring at the lower end of the spectrum), then $\varphi_{\text{SSIP}} = -0.051$ rad, and $\Delta\varphi_{\text{CSIP}} = -0.118$ rad. The resulting peak position variation is -0.13 resolution cell. Although these results appear small, we will demonstrate in Section IV that their effects on the range split-spectrum method can still be significant.

D. Effective Center Frequency under Arbitrary Range Spectral Loss

For the general case of multiple, arbitrarily located lost spectral segments, as shown in Fig. 2, the point target response can be obtained by coherently summing the contributions from

all remaining spectral segments. Although deriving an analytical solution for the final peak position and phase remains challenging due to the different peak positions exhibited by individual segments, we have identified several general behaviors from the theoretical analysis below and the extensive simulations in Section V-A.

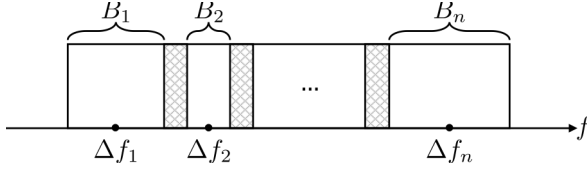


Fig. 2. Illustration of multiple, arbitrarily located lost spectral segments .

In particular, an intuitive approach is to define an effective center frequency weighted according to the bandwidth of each segment:

$$\Delta f_{\text{eff}} = \frac{\sum_{i=1}^n B_i \Delta f_i}{\sum_{i=1}^n B_i} \quad (32)$$

where Δf_i and B_i are the center frequency and bandwidth of the i -th spectral segment, respectively, and n is the total number of remaining spectral segments.

When spectral loss occurs exclusively at the ends of the range spectrum, i.e., the remaining spectrum forms a single continuous segment with a center frequency Δf , both the peak position variation and the SSP within the phase-stable region around the peak position depend on Δf , as given in (19) and (29). For the general case of spectral loss, extensive simulations show that although the exact peak position cannot be predicted analytically, a phase-stable region nevertheless persists around it. Within this region, the phase exhibits an approximately linear variation, analogous to the SSP. In view of this, we further define a peak-position equivalent center frequency and an SSP equivalent center frequency by (19) and (29), corresponding to the peak position variation and the SSP, respectively. The defined effective center frequency Δf_{eff} can accurately represent the SSP equivalent center frequency as demonstrated by the simulations in Section V-A and theoretically justified in Appendix E. It is less accurate for the peak-position equivalent center frequency.

Equation (32) can also be expressed in terms of parameters of the lost spectral segments due to, e.g., the RFI:

$$\Delta f_{\text{eff}} = - \frac{\sum_{i=1}^m B_{\text{RFI}_i} \Delta f_{\text{RFI}_i}}{B - \sum_{i=1}^m B_{\text{RFI}_i}} \quad (33)$$

where Δf_{RFI_i} and B_{RFI_i} are the center frequency and bandwidth of the i -th RFI, respectively, and m is the total number of RFI signals. Specifically, when $m = 1$, i.e., only one RFI is present, (33) can be simplified as

$$\Delta f_{\text{eff}} = - \frac{B_{\text{RFI}}}{B - B_{\text{RFI}}} \Delta f_{\text{RFI}} \quad (34)$$

This indicates that the slope of the SSP within the phase-stable region is approximately linearly dependent on the RFI center frequency, with a negative coefficient determined by the RFI bandwidth.

E. Summary of this Section

Quadrature demodulation and matched filtering eliminate all time-varying characteristics from the phase of the SAR signal, leaving only a phasor representation determined by propagation-related physical processes. Non-dispersive processes involving backscattering, geometry, and the troposphere introduce a frequency-independent position delay given by (7) and a constant phase given by (8). If backscattering is not considered, these two quantities are equal when expressed in the same units. The ionosphere is the sole significant dispersive process considered here, introducing a position delay μ_1 , a constant phase advance DIP, and a time-varying phase SIP that all depend on center frequency, f_c . The position delay and the phase advance are equal in magnitude when expressed in the same units. Within the phase-stable region, the SIP can be approximated by the CSIP, as the VSIP is negligible. The full phase expression is given by (16).

The spectral loss occurring exclusively at the ends of the range spectrum applies to both the wavenumber-shift case and the intentional bandpass filtering case treated later. Two scenarios arise, depending on whether the center frequency is shifted to zero following the spectral loss. In the case where it is, the signal becomes equivalent to the point target response of a new SAR system. Consequently, the full-band results remain applicable, requiring only an update of the carrier frequency and bandwidth parameters. The non-dispersive phase, DIP, and CSIP all vary with the new system parameters, but VSIP remains approximately zero, as indicated by (18). In terms of target position, the non-dispersive delay stays constant, whereas the ionospheric delay varies. Generally, the original peak position remains within the phase-stable region centered around the new peak position.

In the other scenario, the non-zero center frequency due to spectral loss is not shifted to zero. The resulting point target response differs from the previous case only by a linear phase in time, with the target position unchanged. This linear phase cancels the variations in the non-dispersive phase and DIP from the previous case, restoring the phase to that of the original full-band signal. Accordingly, the leftover phase is the SSP, as shown in (25) and (28). The time-domain phase within the phase-stable region can then be expressed as

$$\varphi_{\text{loss}} = \varphi_{\text{non-disp}} + \varphi_{\text{DIP}} + \varphi_{\text{SSP}} + \varphi'_{\text{CSIP}} \quad (35)$$

Table II summarizes the phases for the above scenarios.

The phase-invariant point, where $\varphi_{\text{SSP}} = 0$, approximately coincides with the midpoint between the peak positions before and after range spectral loss. For a non-dispersive system, these three positions coincide, which explains why range spectral loss has no effect on the non-dispersive peak phase. For a dispersive system, these three positions differ; consequently, the SSP terms in the PPV and OPV are approximately equal in magnitude but opposite in sign (i.e., $-\text{SSIP}$ and $+\text{SSIP}$), as shown in (31).

TABLE II
POINT TARGET RESPONSE UNDER DIFFERENT SCENARIOS IN FIG. 1

Scenario	Bandwidth	Carrier frequency	Downconverted center frequency	Non-dispersive phase	DIP	CSIP	VSIP	SSP
1 ^a	B	f_c	0	$\varphi_{bs} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}}$	$\frac{4\pi K_2}{cf_c} \text{TEC}$	$\frac{\pi K_2 B^2}{3cf_c^3} \text{TEC}$	≈ 0	-
2 ^b	B'	$f_c + \Delta f$	Δf	$\varphi_{bs} - 2\pi f_c t_{\text{geom}} - 2\pi f_c t_{\text{tropo}}$	$\frac{4\pi K_2}{cf_c} \text{TEC}$	$\frac{\pi K_2 B'^2}{3c(f_c + \Delta f)^3} \text{TEC}$	≈ 0	$2\pi \Delta f \Delta t$
3 ^c	B'	$f_c + \Delta f$	0	$\varphi_{bs} - 2\pi(f_c + \Delta f)t_{\text{geom}} - 2\pi(f_c + \Delta f)t_{\text{tropo}}$	$\frac{4\pi K_2}{c(f_c + \Delta f)} \text{TEC}$	$\frac{\pi K_2 B'^2}{3c(f_c + \Delta f)^3} \text{TEC}$	≈ 0	-

a Point target response of full band.

b Point target response with range spectral loss, but without shifting center frequency to zero.

c Point target response with range spectral loss, with center frequency shifted to zero.

The generalized case of multiple, arbitrarily located lost spectral segments can be approached via coherently summing the individual point target responses from each segment of the remaining spectrum. Here, we consider only the second case, where the center-frequency shift is not compensated after the loss. The analytical solution is not provided for the peak position of the superposed response. However, a phase-stable region still exists around the peak position, and the linear phase variation within this region can be well predicted by the effective center frequency given by (32).

III. INTERFEROGRAM

A. Signal Model of Distributed Target

Under the first-order Born approximation, the distributed target is modeled as a superposition of numerous non-interacting elementary point targets embedded in a perfectly absorbing background [23]. Assuming that all elementary point targets within a distributed target are subject to identical tropospheric and ionospheric effects, with differences arising solely from geometric range variations, the time-domain signal of the distributed target can be expressed as

$$p_r(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} v_r(u_r) \exp\{-j2\pi(f + f_c)(t_{\text{geom},r} + t_{\text{tropo},r} + u_r)\} H_{\text{iono},r}(f + f_c) \exp\{j2\pi f t\} df du_r \quad (36)$$

where $t_{\text{geom},r}$ is the geometric time delay of the distributed target center, u_r is the relative geometric time delay between an elementary point target and the distributed target center, $t_{\text{tropo},r}$ is the tropospheric time delay, $v_r(u_r)$ is the reflectivity density of the distributed target, $H_{\text{iono},r}(f + f_c)$ is given by (10), and the term $\exp\{j2\pi f t\}$ is the kernel of the inverse Fourier transform that transforms the spectrum in (2) to the time domain. Parameters that vary between acquisitions are distinguished by subscripts, where the subscript 'r' specifically denotes the reference signal, whereas for the secondary signal, the subscript 's' is adopted. The integration limits of u_r are determined by the spatial extent of the distributed target; however, they are assumed to be infinite here for analytical convenience. The implications of this assumption are discussed in Appendix F.

The position of the distributed target center is given by $t = t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}$, where $\mu_{1,r}$ denotes the ionosphere-

induced time delay. Evaluating the signal at this position yields

$$\begin{aligned} z_r &= p_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}) \\ &= \exp\{-j2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r})\} \\ &\quad \cdot \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} v_r(u_r) \exp\{-j2\pi(f + f_c)u_r\} du_r \right\} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} H_{\text{iono},r}(f + f_c) \exp\{j2\pi f \mu_{1,r}\} df \\ &= \exp\{-j2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r})\} \exp\{j\mu_{0,r}\} \\ &\quad \cdot \int_{-\infty}^{+\infty} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} V_r(f + f_c) \exp\{j\mu_{2,r}f^2\} df \end{aligned} \quad (37)$$

where $V_r(f) = \int_{-\infty}^{+\infty} v_r(u_r) \exp\{-j2\pi f u_r\} du_r$ is the Fourier transform of the reflectivity density $v_r(u_r)$.

Since the relative geometric time delay u_r between an elementary point target and the distributed target center is defined along the range direction, it varies with the radar look angle. Therefore, we define an alternative relative geometric time delay along the ground slope as $u = 2\Delta x/c$, where Δx is the distance between the point target and the distributed target center. This definition renders u independent of the radar look angle. The corresponding reflectivity density and its Fourier transform are denoted by $v(u)$ and $V(f)$, respectively. At the distributed target center, given a radar look angle θ_r and a terrain slope α , the local incidence angle is $\theta_r - \alpha$. The relationship between u and u_r is $u_r = u \sin(\theta_r - \alpha)$. Combining with $v_r(u_r)du_r = v(u)du$, we obtain

$$V_r(f) = V(k_r f) \quad (38)$$

where $k_r = \sin(\theta_r - \alpha)$. Therefore, the spectral component of the distributed target at frequency $k_r f$ is observed at a different frequency f by the radar, a phenomenon known as the wavenumber shift [1]. Substituting (38) into (37), z_r becomes

$$\begin{aligned} z_r &= p_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}) \\ &= \exp\{-j2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r})\} \exp\{j\mu_{0,r}\} \\ &\quad \cdot \int_{-\infty}^{+\infty} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} V[k_r(f + f_c)] \exp\{j\mu_{2,r}f^2\} df \end{aligned} \quad (39)$$

The last term in (39) induces a stochastic behavior in the phase of a Single Look Complex (SLC) image.

B. Effects of Range Spectral Loss

Assuming that a range spectral loss of $2\Delta f$ occurs at the lower end of the spectrum, which shifts the center frequency of the remaining spectrum to Δf , the position of the distributed target center is correspondingly altered to $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$, as given in (17). Consequently, the signal in (39) becomes

$$\begin{aligned} z'_r &= p'_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}) \\ &= \exp\{-j2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r})\} \exp\{j\mu_{0,r}\} \\ &\quad \cdot \int_{-\frac{B}{2}+2\Delta f}^{\frac{B}{2}} V[k_r(f \\ &\quad + f_c)] \exp\{j\pi\mu_{2,r}f^2\} \exp\{j2\pi(\mu'_{1,r} \\ &\quad - \mu_{1,r})f\} df \end{aligned} \quad (40)$$

To evaluate the impact of range spectral loss on InSAR, we calculate the expected interferogram $E[z_r(z'_r)^*]$. Assuming that $v(u)$ is stationary and white [24], and following Appendix F, the autocorrelation function of $V(f)$ is given by

$$E[V(f_1)V^*(f_2)] = \sigma^2 \delta(f_2 - f_1) \quad (41)$$

which implies that non-overlapping spectral components do not contribute to $E[z_r(z'_r)^*]$. Therefore,

$$\begin{aligned} E[z_r(z'_r)^*] &= \sigma^2 \int_{-\frac{B}{2}+2\Delta f}^{\frac{B}{2}} \exp\{j2\pi(\mu_{1,r} - \mu'_{1,r})f\} df \\ &= \sigma^2 (B - 2\Delta f) \cdot \text{sinc}[(B - 2\Delta f)(\mu_{1,r} - \mu'_{1,r})] \\ &\quad \cdot \exp\{j2\pi\Delta f(\mu_{1,r} - \mu'_{1,r})\} \end{aligned} \quad (42)$$

The expected phase of the interferogram is derived as

$$\begin{aligned} \text{Arg}\{E[z_r(z'_r)^*]\} &= 2\pi\Delta f(\mu_{1,r} - \mu'_{1,r}) \\ &= 2\pi\Delta f \left[\frac{2K_2}{cf_c^2} \text{TEC}_r - \frac{2K_2}{c(f_c + \Delta f)^2} \text{TEC}_r \right] \\ &\approx \frac{8\pi K_2 \text{TEC}_r}{cf_c^3} (\Delta f)^2 \end{aligned} \quad (43)$$

Equations (39) and (40) show that the interferogram is evaluated at $t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}$ for $p_r(t)$, and at $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$ for $p'_r(t)$. Since the non-overlapping spectrum in $p_r(t)$ does not contribute to the expected interferogram, another scenario is the interferogram evaluated at $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$ for both $p_r(t)$ and $p'_r(t)$. In this case, the interferometric phase calculated using the above method becomes zero. We refer to these two scenarios as coregistration at the actual position and coregistration at the interferometric effective position, respectively. Fig. 3 illustrates these two coregistration schemes, conceptually represented by a point target located at the distributed target center.

According to Section II-C, within the phase-stable region centered around the new distributed target center in the presence of range spectral loss, the time-varying phase, i.e., the SSP, is $2\pi\Delta f\Delta t$. In addition, the original distributed target center lies within this phase-stable region. This explains the difference in the interferometric phase between the two scenarios.

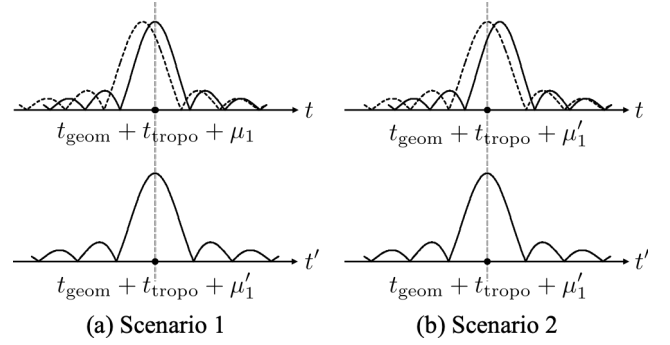


Fig. 3. Two coregistration schemes for computing the interferometric phase. (a) Coregistration at the actual position (Scenario 1). (b) Coregistration at the interferometric effective position (Scenario 2). First row: Original reference signal. Second row: Reference signal with artificially introduced range spectral loss. The solid line denotes the actual position, while the dashed line corresponds to the interferometric effective position. For the original reference signal, the two positions differ; for the reference signal with artificially introduced spectral loss, they coincide.

C. Interferogram in the Presence of Wavenumber Shift

Assume that the secondary signal is acquired with a look angle $\theta_s = \theta_r + \Delta\theta$; then $k_s = \sin(\theta_s - \alpha)$. According to (39), the secondary signal evaluated at the distributed target center is

$$\begin{aligned} z_s &= p_s(t_{\text{geom},s} + t_{\text{tropo},s} + \mu_{1,s}) \\ &= \exp\{-j2\pi f_c(t_{\text{geom},s} + t_{\text{tropo},s})\} \exp\{j\mu_{0,s}\} \\ &\quad \cdot \int_{-\infty}^{+\infty} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} V[k_s(f \\ &\quad + f_c)] \exp\{j\pi\mu_{2,s}f^2\} df \end{aligned} \quad (44)$$

The look angle difference $\Delta\theta$ introduces a relative shift between the spectral components observed by the reference and secondary signals, $V[k_r(f + f_c)]$ and $V[k_s(f + f_c)]$. When the spectra overlap, $k_r(f_r + f_c) = k_s(f_s + f_c)$, and

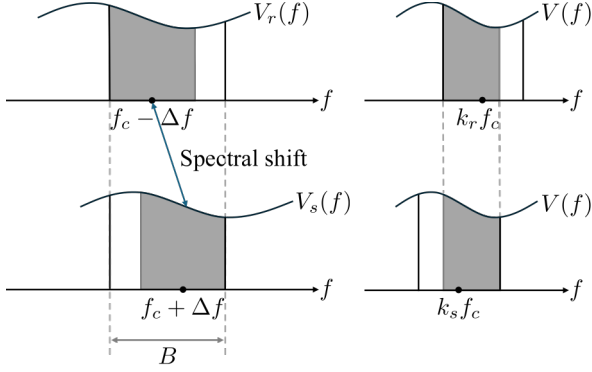
$$f_s = \frac{k_r}{k_s} f_r + \left(\frac{k_r}{k_s} - 1\right) f_c \approx f_r + 2\Delta f \quad (45)$$

where $2\Delta f$ denotes the spectral shift between the two signals [1], and

$$\Delta f = \frac{k_r - k_s}{2k_s} f_c \approx -\frac{\Delta\theta}{2 \tan(\theta_r - \alpha)} f_c \quad (46)$$

Then the center of the overlapping spectrum is at $f_c - \Delta f$ in $V_r(f)$ and at $f_c + \Delta f$ in $V_s(f)$, as shown in Fig. 4. Following quadrature demodulation, the interferometric effective center frequencies of the reference and secondary signals become $-\Delta f$ and Δf , respectively.

It should be noted that range common-band filtering for removing the non-overlapping spectra does not affect the expected interferogram, but rather reduces the noise level of the interferogram. Combining (39), (44), and (F.9), the expected interferogram between the reference and secondary signals can be expressed as



(a) Slant range spectrum (b) Ground range spectrum

Fig. 4. Illustration of spectral shift. First row: The spectrum of reference signal. Second row: The spectrum of secondary signal. The shaded area denotes the overlapping spectral regions.

$$\begin{aligned}
 E[z_r z_s^*] &= \sigma^2 \exp\{-j2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}})\} \exp\{j\Delta\mu_0\} \\
 &\quad \cdot \int_{-\frac{B}{2} - |\Delta f|}^{\frac{B}{2} - |\Delta f|} \exp\{j\pi[\mu_{2,r}(f - \Delta f)^2 \\
 &\quad - \mu_{2,s}(f + \Delta f)^2]\} df \\
 &= \sigma^2 \exp\{-j2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}})\} \exp\{j\Delta\mu_0\} \\
 &\quad \cdot \exp\{j\pi\Delta\mu_2(\Delta f)^2\} \int_{-\frac{B}{2} - |\Delta f|}^{\frac{B}{2} - |\Delta f|} \exp\{j\pi\Delta\mu_2 f^2 \\
 &\quad - j2\pi\Delta f(\mu_{2,r} + \mu_{2,s})f\} df
 \end{aligned} \quad (47)$$

The phase of $E[z_r z_s^*]$ is given by

$$\begin{aligned}
 \Delta\phi_1 &\approx -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \Delta\mu_0 + \pi\Delta\mu_2(\Delta f)^2 \\
 &\quad + \frac{\pi\Delta\mu_2}{3}\left(\frac{B}{2} - |\Delta f|\right)^2 \\
 &= -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c} \\
 &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c^3}(\Delta f)^2 \\
 &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{3cf_c^3}\left(\frac{B}{2} - |\Delta f|\right)^2 \\
 &= \Delta\phi_{\text{non-disp}} + \Delta\phi_{\text{DIP}} + \Delta\phi_{\text{SSIP}} + \Delta\phi'_{\text{CSIP}}
 \end{aligned} \quad (48)$$

Each quantity preceded by Δ in (47)-(48), except Δf , represent the difference in the corresponding variable between the reference and secondary signals. In (48), the contribution of the phase term that varies linearly with frequency f in the integrand of (47) is approximated as zero, since the term is antisymmetric over the symmetric integration interval. A more general and detailed derivation is provided in Appendix G.

The use of $p_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r})$ and $p_s(t_{\text{geom},s} + t_{\text{tropo},s} + \mu_{1,s})$ implies that the two signals are coregistered at their actual positions. The other scenario is coregistration at the interferometric effective positions, which involves performing the complex conjugate multiplication using $p'_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r})$ and $p'_s(t_{\text{geom},s} + t_{\text{tropo},s} + \mu'_{1,s})$. Fig. 5 illustrates these two coregistration schemes, represented by the point target located at the distributed target center. Following a similar derivation, the expected interferometric phase is given by

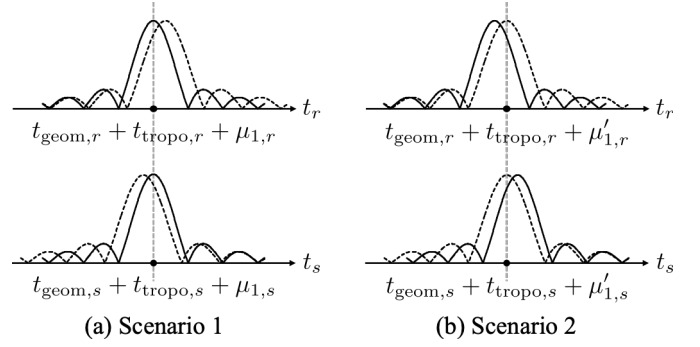


Fig. 5. Two coregistration schemes for computing the interferometric phase. (a) Coregistration at the actual position (Scenario 1); (b) Coregistration at the interferometric effective position (Scenario 2). First row: Reference signal. Second row: Secondary signal. The solid line denotes the actual position, while the dashed line corresponds to the interferometric effective position.

$$\begin{aligned}
 \Delta\phi_2 &\approx -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \Delta\mu_0 - \pi\Delta\mu_2(\Delta f)^2 \\
 &\quad + \frac{\pi\Delta\mu_2}{3}\left(\frac{B}{2} - |\Delta f|\right)^2 \\
 &= -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c} \\
 &\quad - \frac{4\pi K_2 \Delta \text{TEC}}{cf_c^3}(\Delta f)^2 \\
 &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{3cf_c^3}\left(\frac{B}{2} - |\Delta f|\right)^2 \\
 &= \Delta\phi_{\text{non-disp}} + \Delta\phi_{\text{DIP}} - \Delta\phi_{\text{SSIP}} + \Delta\phi'_{\text{CSIP}}
 \end{aligned} \quad (49)$$

From (48) and (49), it can be observed that the interferometric non-dispersive phase is independent of Δf and is therefore unaffected by spectral loss. The interferometric ionospheric phase consists of three components: the Differential Dominant Ionospheric Phase (DDIP), the Differential Spectral-Shifting Ionospheric Phase (DSSIP), and the Differential Constant Secondary Ionospheric Phase (DCSIP). The DSSIP depends on the chosen coregistration scheme. During interferogram formation, non-overlapping spectra can be considered “lost”. When coregistration is performed at the actual position, it corresponds to the effect of range spectral loss on the phase at the original peak position described in Section II-C, i.e., the ϕ_{SSIP} in $\phi_{\text{peak,ori}}$ in (31). Conversely, coregistration at the interferometric effective position corresponds to the $-\phi_{\text{SSIP}}$ in $\phi_{\text{peak,new}}$.

The above results indicate that the interferometric phase of a distributed target can be consistently described using a point target model, provided that the range spectral loss induced by wavenumber shift is incorporated into the point target analysis framework, although such a spectral loss does not naturally occur for point targets.

D. Phase Error due to Coregistration Misalignment

In Section III-C, we defined two coregistration schemes: the first based on the actual position and the second based on the interferometric effective position. If the cross-correlation method is employed to estimate the range offset, the non-

overlapping spectra caused by wavenumber shift are uncorrelated for distributed targets; therefore, they do not contribute to the expected cross-correlation offset. However, for point targets, the expected cross-correlation offset is unaffected by the wavenumber shift effect. Consequently, point targets are coregistered at their actual positions, whereas distributed targets are coregistered at their interferometric effective positions. Furthermore, if multiple secondary signals are present, Δf differs among interferometric pairs, leading to varying interferometric effective positions for the same target. In the subsequent discussion, we adopt the more intuitive first coregistration scheme as the reference.

In practical data processing, the actual target position may deviate from the nominal position, leading to a residual phase error after topographic phase removal, as shown in Fig. 6. Therefore, when analyzing coregistration misalignment after resampling of the secondary signal, three distinct positions need to be considered. The first is the actual position in the reference signal, denoted by A in Fig. 6. The second is the actual position in the resampled secondary signal, denoted by A' . The third is the nominal position, denoted by B , i.e., the position assumed in the processing, at which the geometric phase difference corresponding to actual positions A and A' is removed for topographic phase correction. Here, $\Delta\tau_r$ and $\Delta\tau_s$ represent the misalignments of positions A and A' relative to position B in the reference and secondary signals, respectively, hereafter referred to as the reference misalignment and secondary misalignment. At position B , the interferogram is formed by the complex conjugate multiplication of $p_r(t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r} + \Delta\tau_r)$ and $p_s(t_{\text{geom},s} + t_{\text{tropo},s} + \mu_{1,s} + \Delta\tau_s)$, and the corresponding interferometric phase contains an additional misalignment phase, augmenting the phase defined in (48):

$$\varphi_{\text{mis}} = -2\pi\Delta f(\Delta\tau_r + \Delta\tau_s) \quad (50)$$

A detailed derivation is provided in Appendix G.

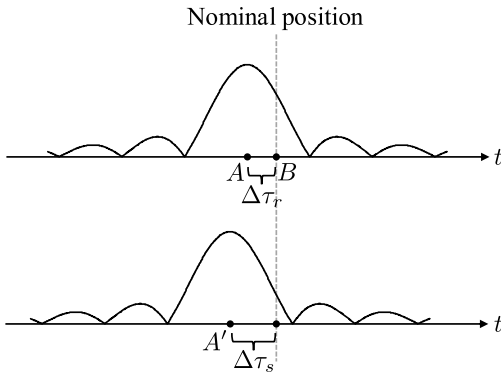


Fig. 6. Illustration of coregistration misalignment.

The point target model remains applicable for interpreting the misalignment phase in (50). After excluding the non-overlapping spectra caused by wavenumber shift, the interferometric effective center frequencies are $-\Delta f$ for the reference signal and Δf for the secondary signal. This introduces the SSP, i.e., linear time-varying phases with slopes

$-2\pi\Delta f$ and $2\pi\Delta f$, into the corresponding point target responses. Consequently, the reference and secondary misalignments introduce phase errors of $-2\pi\Delta f\Delta\tau_r$ and $2\pi\Delta f\Delta\tau_s$, respectively, and their difference yields the phase term in (50).

If $\Delta\tau_r = \Delta\tau_s$, the actual positions in the reference and secondary signals are aligned after resampling. The remaining misalignment only exists between the actual and nominal positions, hereafter referred to as the absolute misalignment. As shown in Fig. 7, the resulting phase error arises from the difference in the interferometric geometric phase between A and B :

$$\begin{aligned} \varphi_{\text{mis}} &= -2\pi f_c \frac{2(|RB| - |SB|)}{c} + 2\pi f_c \frac{2(|RA| - |SA|)}{c} \\ &= -2\pi f_c \left[\frac{2(|RB| - |RA|)}{c} - \frac{2(|SB| - |SA|)}{c} \right] \\ &= -2\pi f_c \frac{2(|RB| - |RA|)}{c} \left[1 - \frac{\sin(\theta_s - \alpha)}{\sin(\theta_r - \alpha)} \right] \\ &= -2\pi f_c \Delta\tau_r \frac{k_r - k_s}{k_r} \\ &= -4\pi\Delta f\Delta\tau_r \end{aligned} \quad (51)$$

which is consistent with (50).

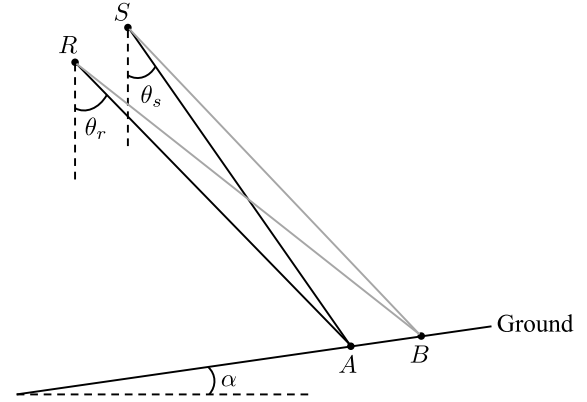


Fig. 7. Illustration of absolute misalignment.

If $\Delta\tau_r = 0$, the misalignment exists only between the reference and secondary signals, hereafter referred to as the relative misalignment. In this scenario, $\varphi_{\text{mis}} = -2\pi\Delta f\Delta\tau_s$, which is consistent with the result reported in [25]. Note that the definitions of Δf and $\Delta\tau_s$ in this paper differ from those in [25], leading to a factor of -2 in φ_{mis} . For the general case where $\Delta\tau_r \neq 0$, the phase error due to relative misalignment becomes $\varphi_{\text{mis}} = -2\pi\Delta f(\Delta\tau_s - \Delta\tau_r)$. Equation (50) can also be reformulated as $\varphi_{\text{mis}} = -4\pi\Delta f\Delta\tau_r - 2\pi\Delta f(\Delta\tau_s - \Delta\tau_r)$, implying that coregistration misalignment can be decomposed not only into reference and secondary misalignments, but also into absolute and relative misalignments.

According to the results in Section II, when considering the spectral loss induced by wavenumber shift, the phase-invariant point in the point target response is the midpoint between the actual position and the interferometric effective position. If this phase-invariant point is regarded as the true target position, the DSSIP terms in (48) and (49) can also be interpreted in terms of coregistration misalignment. This

coregistration-misalignment perspective will subsequently be employed to explain the phase errors introduced in ionospheric phase estimation using the range split-spectrum method.

E. Summary of this Section

The signal model for distributed targets is developed based on the superposition of numerous non-interacting elementary point targets, and is further formulated in terms of the reflectivity density spectrum, as given in (39). This approach not only provides a perspective for understanding the stochastic phase behavior of SAR signals and the wavenumber shift effect, but also provides a means to compute the interferometric phase under spectral-loss conditions.

For distributed targets, wavenumber shift induces range spectral loss in the interferometric process, leading to interferometric effective center frequencies of $-\Delta f$ and Δf for the reference and secondary signals, respectively. The resulting interferometric phase is consistent with that obtained from the point target model accounting for the spectral loss.

It should be noted that, when considering the spectral loss induced by wavenumber shift, the center frequency is not shifted to zero after the loss. In the corresponding point target model, the non-dispersive phase and DIP are unaffected by the spectral loss, but the CSIP varies with the reduced bandwidth, and an additional time-varying phase, the SSP, emerges. Accordingly, as given by (48) and (49), the range spectral loss induced by wavenumber shift has no effect on the non-dispersive components of the interferometric phase. The interferometric ionospheric phase, in contrast, is affected. It comprises the DDIP, the DCSIP, and the DSSIP. The DCSIP depends on the overlapping bandwidth $B - 2|\Delta f|$ rather than the full bandwidth B . The DSSIP depends on the coregistration scheme. For coregistrations based on the actual position and the interferometric effective position, the DSSIP corresponds to the SSIP evaluated at the original peak position and the new peak position after spectral loss in (31), respectively. Following the numerical example in Section II-C, both the DSSIP and DCSIP have negligible magnitudes. Relative to the DDIP, the magnitudes of the DSSIP and DCSIP are reduced by factors of $(\Delta f/f_c)^2$ and $(B - 2|\Delta f|)^2/12f_c^2$, respectively. Given that $\Delta f \ll f_c$ and $B \ll f_c$, the DDIP dominates the total interferometric ionospheric phase. Consequently, correcting the DDIP alone is generally sufficient for InSAR ionospheric phase correction.

Coregistration misalignment, which encompasses reference and secondary misalignments, introduces additional phase errors, as given by (50), arising from the SSP, i.e., the linear time-varying phase introduced by spectral loss. Coregistration misalignment can also be interpreted in terms of absolute and relative misalignments, yielding the same misalignment phase. The coregistration-misalignment perspective provides a unified framework for interpreting the interferometric phase errors in various scenarios, including the DSSIP.

IV. RANGE SPLIT-SPECTRUM METHOD

A. Sub-band Interferometric Phase

The range split-spectrum method generates two or more sub-bands through bandpass filtering in the range-frequency domain and has been used to determine the absolute InSAR phase [14], estimate the residual range offset between SAR image pairs [15] (known as the spectral-diversity method), and estimate the InSAR ionospheric phase [16]. However, certain variations in ionospheric effects introduced by sub-band extraction have been overlooked in previous research. For example, since the ionosphere-induced position delay depends on the center frequency, extracting a sub-band shifts the target position relative to the full-band response.

Assuming that the full-band reference and secondary images are coregistered at the actual position, we consider a sub-band with center frequency f_{sub} and bandwidth B_{sub} . Strictly speaking, the range spectral shift between the sub-band reference and secondary images, arising from the wavenumber shift effect, is $2\Delta f \cdot f_c/f_{\text{sub}}$, where $2\Delta f$ is the spectral shift of the full band. Nonetheless, the full-band spectral shift $2\Delta f$ is retained as an approximation. Accounting for quadrature demodulation and the range spectral shift, the interferometric effective center frequency is $f_{\text{sub}} - f_c - \Delta f$ for the reference image, and $f_{\text{sub}} - f_c + \Delta f$ for the secondary image. We first consider the scenario in which the center frequency $f_{\text{sub}} - f_c$ is not shifted to zero after bandpass filtering. As established in Section III, the interferometric phase of distributed targets can be derived using the point target model. Following (31), the phases at the original full-band peak positions in the sub-band images are given by

$$\begin{aligned} \varphi_{\text{peak,ori},r} = & \varphi_{\text{bs}} - 2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r}) + \frac{4\pi K_2}{cf_c} \text{TEC}_r \\ & + \frac{4\pi K_2 \text{TEC}_r}{cf_c^3} (f_{\text{sub}} - f_c - \Delta f)^2 \\ & + \frac{4\pi K_2 \text{TEC}_r}{3cf_{\text{sub}}^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f| \right)^2 \end{aligned} \quad (52)$$

$$\begin{aligned} \varphi_{\text{peak,ori},s} = & \varphi_{\text{bs}} - 2\pi f_c(t_{\text{geom},s} + t_{\text{tropo},s}) + \frac{4\pi K_2}{cf_c} \text{TEC}_s \\ & - \frac{4\pi K_2 \text{TEC}_s}{cf_c^3} (f_{\text{sub}} - f_c + \Delta f)^2 \\ & + \frac{4\pi K_2 \text{TEC}_s}{3cf_{\text{sub}}^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f| \right)^2 \end{aligned}$$

Then the interferometric phase is directly derived as

$$\begin{aligned} \Delta\varphi_{\text{sub},1} = & \varphi_{\text{peak,ori},r} - \varphi_{\text{peak,ori},s} \\ = & -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c} \\ & + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c^3} [(f_{\text{sub}} - f_c)^2 + (\Delta f)^2] \\ & + \frac{4\pi K_2 \sum \text{TEC} \cdot \Delta f \cdot 2(f_c - f_{\text{sub}})}{cf_c^3} \\ & + \frac{4\pi K_2 \Delta \text{TEC}}{3cf_{\text{sub}}^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f| \right)^2 \end{aligned} \quad (53)$$

where $\sum \text{TEC} = \text{TEC}_r + \text{TEC}_s$.

In the second scenario, the center frequency $f_{\text{sub}} - f_c$ is shifted to zero after bandpass filtering by applying a linear phase of $2\pi(f_c - f_{\text{sub}})t$ in the time domain. Consequently, the

interferometric phase becomes

$$\begin{aligned}\Delta\varphi_{\text{sub},2} &= [\varphi_{\text{peak,ori},r} + 2\pi(f_c - f_{\text{sub}}) \\ &\quad \cdot (t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r})] \\ &\quad - [\varphi_{\text{peak,ori},s} + 2\pi(f_c - f_{\text{sub}}) \\ &\quad \cdot (t_{\text{geom},s} + t_{\text{tropo},s} + \mu_{1,s})] \\ &\approx -2\pi f_{\text{sub}}(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{cf_{\text{sub}}} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{cf_c^3} (\Delta f)^2 \\ &\quad + \frac{4\pi K_2 \sum \text{TEC} \cdot \Delta f \cdot 2(f_c - f_{\text{sub}})}{cf_c^3} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{3cf_{\text{sub}}^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f| \right)^2\end{aligned}\quad (54)$$

The interferometric phases for both scenarios can also be derived following the approach described in Appendix G. If we treat the second scenario as an equivalent SAR system characterized by the corresponding sub-band parameters, (54) can be obtained directly from (48) by replacing f_c and B with f_{sub} and B_{sub} , respectively. However, (54) contains an additional phase term

$$\frac{4\pi K_2 \sum \text{TEC} \cdot \Delta f \cdot 2(f_c - f_{\text{sub}})}{cf_c^3} \quad (55)$$

This term also appears in (53). It originates from the difference between the actual positions of the sub-band and full-band SAR systems. In practical data processing, the positional difference is typically neglected, with the latter being regarded as the nominal position. Therefore, this phase term arises from the coregistration misalignment, which is attributable to the target position variation induced by sub-band filtering. Following (19), the reference and secondary misalignments are given by

$$\begin{aligned}\Delta\tau_r &= \frac{4K_2(f_{\text{sub}} - f_c)}{cf_c^3} \text{TEC}_r \\ \Delta\tau_s &= \frac{4K_2(f_{\text{sub}} - f_c)}{cf_c^3} \text{TEC}_s\end{aligned}\quad (56)$$

Therefore, according to (50), their impact on the interferometric phase is directly derived as

$$\begin{aligned}\varphi_{\text{mis}} &= -2\pi\Delta f(\Delta\tau_r + \Delta\tau_s) \\ &= \frac{4\pi K_2 \sum \text{TEC} \cdot \Delta f \cdot 2(f_c - f_{\text{sub}})}{cf_c^3}\end{aligned}\quad (57)$$

which is equivalent to (55).

B. InSAR Ionospheric Phase Estimation

Employing the range split-spectrum method, the InSAR ionospheric phase can be estimated [16]. To achieve the Cramér–Rao Lower Bound (CRLB), the full band is typically partitioned into three sub-bands of equal bandwidth, with the first and third used for estimation [26]. Under this framework, the InSAR ionospheric phase of the full band is estimated. This split-spectrum scheme is referred to as the trisecting scheme. Newer SAR missions, such as ALOS-4 [27] and NISAR [28], incorporate an additional sideband alongside the main band to enhance the estimation precision. In this configuration, the range split-spectrum method is applied to estimate the InSAR ionospheric phase of the main band. This scheme is referred to as the main-sideband scheme.

We begin with the scenario of a full band partitioned via the trisecting scheme. Subsequently, the ionospheric phase in the full-band interferogram can be derived as

$$\Delta\varphi_{\text{iono}} = \frac{f_l f_u}{f_c(f_u^2 - f_l^2)} (f_u \Delta\varphi_l - f_l \Delta\varphi_u) \quad (58)$$

where f_l and f_u are the center frequencies of the lower and upper sub-bands, and $\Delta\varphi_l$ and $\Delta\varphi_u$ are the corresponding sub-band interferometric phases.

In InSAR ionospheric phase estimation, the center frequency of each sub-band should be shifted to zero. Therefore, (54) should be used to calculate the sub-band interferometric phases. By substituting f_{sub} and B_{sub} with $f_c \pm B/3$ and $B/3$, respectively, $\Delta\varphi_l$ and $\Delta\varphi_u$ can be obtained. The phase error in (57) is present in both interferograms. Since $f_c - f_l$ and $f_c - f_u$ are opposite in sign, the phase errors corresponding to the lower and upper sub-bands are also opposite in sign, causing the errors to be amplified when the two sub-band phases are combined in (58). In the presence of these errors, the estimated ionospheric phase becomes

$$\begin{aligned}\Delta\varphi_{\text{iono}} &\approx \frac{4\pi K_2 \Delta \text{TEC}}{cf_c} + 4\pi K_2 \frac{\sum \text{TEC} \Delta f}{cf_c^2} + \frac{2\pi K_2 \Delta \text{TEC} (\Delta f)^2}{cf_c^3} \\ &\quad + \frac{8\pi K_2 \Delta \text{TEC} \left(\frac{B}{6} - \Delta f \right)^2}{cf_c^3}\end{aligned}\quad (59)$$

A comparison with the ionospheric phase in (48) reveals that:

- (1) The first term in (59) corresponds to the desired DDIP component to be corrected.
- (2) The second term represents the phase error introduced by (57) and is termed the Position Variation Phase (PVP). It is expressed as

$$\varphi_{\text{PVP}} = 4\pi K_2 \frac{\sum \text{TEC} \Delta f}{cf_c^2} \quad (60)$$

- (3) The remaining terms originate from the DSSIP and the DCSIP components of the sub-band interferograms.

Since the DSSIP and DCSIP are generally negligible, the dominant error is the PVP.

As proposed in [29], to improve robustness, (58) can be reformulated as

$$\Delta\varphi_{\text{iono}} = \frac{f_l f_u}{f_c(f_u + f_l)} \Delta\varphi_0 + \frac{f_l f_u}{2f_c(f_u - f_l)} (\Delta\varphi_l - \Delta\varphi_u) \quad (61)$$

where $\Delta\varphi_0$ is the full-band interferometric phase. Substituting (54) into (61) shows that the resulting PVP is identical to that given in (60).

Assuming $\text{TEC}_r = \text{TEC}_s = 60 \text{ TECU}$ and $\Delta f = 1 \text{ MHz}$, the PVP is approximately 1.3 rad for L-band. PVP depends not only on the sum of the TEC values, $\sum \text{TEC}$, but also on the range spectral shift, Δf . In wide-swath modes, Δf can vary significantly from near range to far range, leading to a range-varying PVP. When $\sum \text{TEC}$ and the perpendicular baseline are large, PVP can induce a range phase ramp that cannot be neglected when measuring long-wavelength tectonic motions.

In the second scenario of the main-sideband scheme, the interferometric phase of the main band can be expressed by (48). Assuming that the sideband has a center frequency $f_{\text{side}} = f_c + f_{\text{sep}}$ and a bandwidth B_{side} , its interferometric phase can be obtained from (54) as

$$\Delta\varphi_{\text{side}} = -2\pi f_{\text{side}}(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta\text{TEC}}{cf_{\text{side}}} - \frac{8\pi K_2 \sum\text{TEC} \Delta f \cdot f_{\text{sep}}}{cf_c^3} + \frac{4\pi K_2 \Delta\text{TEC}(\Delta f)^2}{cf_c^3} + \frac{4\pi K_2 \Delta\text{TEC}}{3cf_{\text{side}}^3} \left(\frac{B_{\text{side}}}{2} - \Delta f \right)^2 \quad (62)$$

The ionospheric phase in the main-band interferogram can then be derived as

$$\Delta\varphi_{\text{iono}} = \frac{f_{\text{side}}}{f_{\text{side}}^2 - f_c^2} (f_{\text{side}} \Delta\varphi_{\text{main}} - f_c \Delta\varphi_{\text{side}}) \approx \frac{4\pi K_2 \Delta\text{TEC}}{cf_c} + 4\pi K_2 \frac{\sum\text{TEC} \Delta f}{cf_c^2} + \frac{2\pi K_2 \Delta\text{TEC}(\Delta f)^2}{cf_c^3} + \frac{4\pi K_2 \Delta\text{TEC} (B + B_{\text{side}} - 4\Delta f)(B - B_{\text{side}})}{cf_c 24f_c f_{\text{sep}}} \quad (63)$$

The first three terms in (63) also appear in (59). The last term originates from the asymmetric bandwidths of the main band and the sideband, which lead to different DCSIP values. It is termed the Bandwidth Variation Phase (BVP). The BVP consists of a Δf -dependent component and a Δf -independent component. Given that $B + B_{\text{side}}$ is typically much greater than Δf , the Δf -independent component constitutes the primary source of error. Furthermore, taking NISAR's 20+5 MHz, 40+5 MHz, and ALOS-4's 28+10 MHz modes as examples, all of them satisfy $B - B_{\text{side}} < f_{\text{sep}}$. Given that $|\Delta\text{TEC}| < \sum\text{TEC}$, the Δf -dependent component is generally an order of magnitude smaller than the PVP. For simplicity, the BVP can therefore be assumed to contain only the Δf -independent component

$$\varphi_{\text{BVP}} = \frac{4\pi K_2 \Delta\text{TEC}}{cf_c} \cdot \frac{B^2 - B_{\text{side}}^2}{24f_c f_{\text{sep}}} = \frac{B^2 - B_{\text{side}}^2}{24f_c f_{\text{sep}}} \varphi_{\text{DDIP}} \quad (64)$$

From (64), the BVP is approximately proportional to the DDIP. Among the three aforementioned operating modes, NISAR's 40+5 MHz mode results in the largest proportionality coefficient, which is only approximately 0.001. For $\Delta\text{TEC} = 60$ TECU, the DDIP is around 806 rad, and the resulting BVP is only 0.8 rad. Furthermore, since InSAR measures relative phase changes and the ionosphere is a spatially slowly varying medium, the impact of BVP is minimal. In practice, the variation in DDIP within a single SAR scene rarely exceeds 100 cycles. The corresponding variation in BVP across the scene is less than 0.1 cycle (0.3 rad). Consequently, the BVP can generally be neglected for L-band systems. Accounting for this term becomes essential only in scenarios characterized by exceptionally large swath widths or severe ionospheric disturbances.

C. Remarks on the PVP

In both the trisecting and main-sideband schemes, the analytical expression for PVP remains identical. It is independent of the frequency separation between the two sub-bands. The coupling of $\sum\text{TEC}$ and Δf may give rise to a non-zero closure phase in any closed loop of interferograms. Given three SAR images indexed by i, j , and k , we can form three interferograms, I_{ij} , I_{jk} and I_{ik} . The closure phase induced by the PVP is given by

$$\Delta\varphi_{\text{PVP}_{ijk}} = \varphi_{\text{PVP}_{ij}} + \varphi_{\text{PVP}_{jk}} - \varphi_{\text{PVP}_{ik}} = \frac{4\pi K_2}{cf_c^2} [\sum\text{TEC}_{ij} \Delta f_{ij} + \sum\text{TEC}_{jk} \Delta f_{jk} - \sum\text{TEC}_{ik} \Delta f_{ik}] = \frac{4\pi K_2}{cf_c^2} [\Delta\text{TEC}_{jk} \Delta f_{ij} - \Delta\text{TEC}_{ij} \Delta f_{jk}] \quad (65)$$

where the sum of range spectral shifts satisfies a closure condition of $\Delta f_{ij} + \Delta f_{jk} - \Delta f_{ik} = 0$. It should be noted that the PVP is a phase error introduced during ionospheric phase estimation. It does not explain the origin of non-zero closure phases in the uncorrected interferograms, but rather contributes to the non-zero closure phases observed after ionospheric phase correction.

It is also instructive to examine how the PVP affects InSAR time-series analysis. First, $\sum\text{TEC}$ is strongly correlated with time. Second, Δf is a function of radar look angle and perpendicular baseline. This typically causes the PVP to manifest as a range phase ramp, particularly in wide-swath modes. For satellites with well-controlled orbits, the perpendicular baseline is generally not correlated with time. Consequently, the PVP does not accumulate systematically over time, and its impact is negligible for InSAR time series generated from a sufficiently large stack of SAR images. However, if the orbit systematically drifts over time, as observed with ALOS-1 [30], [31], the PVP can significantly bias the estimated time series. A detailed derivation is provided in Appendix H.

D. Absolute InSAR Phase Determination

Employing the range split-spectrum method, the absolute InSAR phase can be determined [14], [32]. It is primarily employed in single-pass InSAR systems, including airborne and spaceborne platforms, for generating the Digital Elevation Models (DEMs). In such scenarios, the phase terms associated with target backscattering properties, tropospheric delays, and ionospheric advances cancel out in the interferometric phase, leaving only the phase term due to geometric delays.

Assume that the two sub-bands have center frequencies of f_l and f_u and share a common bandwidth B_{sub} . The center frequency of each sub-band is shifted to zero. The absolute InSAR phase can be derived as

$$\Delta\varphi_{\text{absolute}} = \frac{f_c}{f_u - f_l} (\Delta\varphi_u - \Delta\varphi_l) \quad (66)$$

As the center frequencies of the sub-bands are shifted to zero, the sub-band interferometric phases $\Delta\varphi_l$ and $\Delta\varphi_u$ should be calculated using (54). In the airborne scenario, there are no TEC-related terms. However, in the spaceborne scenario, although $\Delta\text{TEC} = 0$, the sub-band interferometric phases still contain terms related to $\sum\text{TEC}$, which induce a phase error in (66). Consequently, the estimated absolute InSAR phase becomes

$$\Delta\varphi_{\text{absolute}} = -2\pi f_c (\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) - \frac{8\pi K_2 \cdot \sum\text{TEC} \cdot \Delta f}{cf_c^2} \quad (67)$$

The phase error in (67) is twice the PVP, but with the opposite sign. The coefficient associated with the PVP stems from the difference between (58) and (66). Assuming a perpendicular baseline of 1500 m, a look angle of 30°, flat

terrain ($\alpha = 0$), and other parameters from [5], Δf in (46) can reach approximately 2.3 MHz. If $\text{TEC}_r = \text{TEC}_s = 60 \text{ TECU}$, the error in the estimated absolute phase is approximately 5.9 rad. If we use the estimated absolute phase to determine the integer number of 2π in the wrapped phase $\Delta\varphi_{\text{wrapped}}$, this integer can be calculated as

$$m = \text{round}\left\{\frac{\Delta\varphi_{\text{wrapped}} - \Delta\varphi_{\text{absolute}}}{2\pi}\right\} \quad (68)$$

Given the magnitude of the error in (67), there is a low probability that the estimated m is corrupted. If the estimated absolute phase is directly converted to a DEM, then the corresponding height error in the resulting DEM is

$$\begin{aligned} h_{\text{error}} &= \frac{cR \sin \theta_r}{4\pi f_c B_{\perp}} \Delta\varphi_{\text{error}} \\ &= -\frac{2K_2 R \sin \theta_r}{f_c^3 B_{\perp}} \cdot \Sigma \text{TEC} \cdot \Delta f \\ &= \frac{K_2 \sin \theta_r}{f_c^2 \tan(\theta_r - \alpha)} \cdot \Sigma \text{TEC} \end{aligned} \quad (69)$$

where $\Delta\varphi_{\text{error}}$ is the second phase term in (67), $B_{\perp}$ is the perpendicular baseline, and R is the slant range. Remarkably, h_{error} is independent of the perpendicular baseline. Under the aforementioned imaging geometry and ionospheric conditions, the resulting DEM consistently exhibits a height error of 26.4 m, regardless of variations in the perpendicular baseline.

E. Residual Range Offset Estimation

In the previously mentioned applications of the range split-spectrum method, it is assumed that the original full band is free of coregistration misalignment. Now assume that there is no absolute misalignment but only a relative misalignment $\Delta\tau$, i.e., $\Delta\tau_r = 0$ and $\Delta\tau_s = \Delta\tau$, in the original full band. Employing the range split-spectrum method, the residual range offset $\Delta\tau$ between the reference and secondary images [15] can be estimated by

$$\Delta\tau = -\frac{\Delta\varphi_u - \Delta\varphi_l}{2\pi(f_u - f_l)} \quad (70)$$

This method is commonly referred to as the spectral-diversity method. The full band is typically partitioned using the trisecting scheme. The center frequency of each sub-band is not shifted to zero after bandpass filtering. Therefore, (53) should be used to calculate the sub-band interferometric phases. As established in Section III-D, the residual range offset induces a phase term of $-2\pi\Delta f\Delta\tau$ in the interferometric phase of the original full band. Since the sub-band center frequency is shifted from 0 to $f_{\text{sub}} - f_c$, the induced phase term in the sub-band interferometric phase is expressed as $-2\pi(f_{\text{sub}} - f_c + \Delta f)\Delta\tau$. A detailed derivation is provided in Appendix G.

In the trisecting scheme, the two sub-bands are characterized by a common bandwidth B_{sub} and symmetric center frequencies, such that $f_l + f_u = 2f_c$. The resulting interferometric phases of the sub-bands are given by

$$\begin{aligned} \Delta\varphi_l &= -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{c f_c} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{c f_c^3} [(f_l - f_c)^2 + (\Delta f)^2] \\ &\quad + \frac{4\pi K_2 \Sigma \text{TEC} \cdot \Delta f \cdot 2(f_c - f_l)}{c f_c^3} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{3c f_l^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f|\right)^2 \\ &\quad - 2\pi(f_l - f_c + \Delta f)\Delta\tau \\ \Delta\varphi_u &= -2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}}) + \frac{4\pi K_2 \Delta \text{TEC}}{c f_c} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{c f_c^3} [(f_u - f_c)^2 + (\Delta f)^2] \\ &\quad + \frac{4\pi K_2 \Sigma \text{TEC} \cdot \Delta f \cdot 2(f_c - f_u)}{c f_c^3} \\ &\quad + \frac{4\pi K_2 \Delta \text{TEC}}{3c f_u^3} \left(\frac{B_{\text{sub}}}{2} - |\Delta f|\right)^2 \\ &\quad - 2\pi(f_u - f_c + \Delta f)\Delta\tau \end{aligned} \quad (71)$$

Substituting (71) into (70) and neglecting the difference in the DCSIP between $\Delta\varphi_l$ and $\Delta\varphi_u$, the estimated offset, in units of range resolution cells, is given by

$$\Delta x_{\text{estimated}} = B\Delta\tau_{\text{estimated}} \approx B\Delta\tau + 4K_2 \frac{B\Sigma \text{TEC}\Delta f}{c f_c^3} \quad (72)$$

The estimation error of the range offset equals the PVP scaled by $B/\pi f_c$, a coefficient that originates from the difference between (58) and (70). Assuming $\text{TEC}_r = \text{TEC}_s = 60 \text{ TECU}$ and $\Delta f = 1 \text{ MHz}$, the estimation error is approximately 0.026 resolution cell for the ALOS-2 UBS mode. Note that the bandwidth of the ALOS-2 UBS mode is approximately 80 MHz, occupying nearly the entire 85 MHz spectrum allocated for active remote sensing at L-band. Therefore, the resulting error is negligible for L-band systems.

It is worth noting that a residual range offset $\Delta\tau$ induces a phase error of $-2\pi\Delta f\Delta\tau$, which depends solely on the spectral shift arising from wavenumber shift and persists even in the absence of the ionosphere. This phase term was not considered in previous studies, and its potential impact on the estimation of $\Delta\tau$ in (70) was therefore overlooked. Nevertheless, this error is present in both sub-band interferograms. If the difference in Δf between the lower and upper bands is neglected, this error completely cancels out in $\Delta\varphi_u - \Delta\varphi_l$ and does not affect the estimated range offset.

F. Summary of this Section

In the range split-spectrum method, two sources of spectral loss must be distinguished: one stemming from intentional bandpass filtering, and the other arising from wavenumber shift. In the context of ionospheric phase estimation, the non-zero center frequency due to bandpass filtering is typically shifted back to zero, and the complex conjugate product of the filtered data is treated as the interferogram of an equivalent SAR system with the corresponding sub-band parameters. This standard practice, however, does not account for two important factors: the spectral loss induced by wavenumber shift and the target position variation resulting from sub-band filtering.

These two issues can be assessed in the sub-band interferograms. As shown in Section III, wavenumber shift modifies the DCSIP through bandwidth reduction and induces the DSSIP; both phase components are small and largely cancel out when combining the sub-band interferograms for implementing the range split-spectrum method. Meanwhile, the target position in the full-band image is typically taken as the nominal position for the sub-bands; consequently, the target position variation resulting from sub-band filtering manifests as a coregistration misalignment in the sub-band interferograms. In the presence of wavenumber shift, this coregistration misalignment yields an additional phase, as given by (57). Although small by itself, this phase is greatly amplified when combining the sub-band interferograms, leading to the PVP error in the ionospheric phase estimate, as given by (60). Notably, the PVP is independent of the frequency separation between the two sub-bands.

The PVP is appreciable under large perpendicular baselines or strong ionospheric activity. It generally manifests as a range-dependent trend and can introduce closure phases for interferograms with ionospheric phase corrected, particularly in wide-swath modes. Moreover, if the satellite orbit exhibits a systematic drift over time, the PVP may substantially bias InSAR time-series analysis. Similar errors are also present in other applications of the range split-spectrum method, such as absolute InSAR phase determination and residual range offset estimation, as given by (67) and (72).

When the two subbands have different bandwidths, as in the main-sideband scheme, the DCSIP is not canceled in the subband combination and becomes amplified, resulting in the BVP error in the ionospheric phase estimate, as given by (64). The BVP is approximately proportional to the DDIP, with the proportionality coefficient typically less than 0.001, and thus can generally be neglected.

V. SIMULATION

We perform simulations using the SAR system parameters of the ALOS-2 UBS mode, as summarized in Table III. We simulate the complete range-direction SAR imaging process, encompassing signal transmission, propagation, reception, demodulation, and range compression via matched filtering. Note that a range-direction simulation alone is sufficient, as the azimuth direction is irrelevant to the analysis.

TABLE III
SYSTEM PARAMETERS FROM ALOS-2 UBS MODE

Parameter	Value
Center Frequency [MHz]	1257.5
Bandwidth [MHz]	79.4
Range Sampling Rate [MHz]	104.8
Pulse Duration [μ s]	61.2
Look Angle [deg]	36.2
Slant Range at Scene Center [m]	767574.5
Ground Surface Slope [deg]	5.0

The ground surface is assumed to exhibit a uniform slope. The scatterers are randomly distributed along the slope, with their reflectivities following a zero-mean circular complex Gaussian distribution. In total, 500 range pixels are generated. Given the small spatial extent, the Earth curvature is neglected in the imaging geometry. The ionospheric effect is incorporated by applying the ionospheric transfer function given in (10) to the SAR signal in the range-frequency domain. The tropospheric effect is not considered.

For a spatially constant TEC, we first coherently sum the echo signals from all scatterers and then apply the ionospheric effect to the resulting composite SAR signal. For a range-varying TEC, the ionospheric effect must be applied to the echo signal of each individual scatterer before coherent summation.

A. Point Target Response

For point target imaging, we retain only a single scatterer on the ground and assume a reflectivity of 1 ($\varphi_{bs} = 0$) for simplicity. A TEC of 60 TECU is applied to the simulated echo signal. A range spectral loss of $2\Delta f = 20$ MHz occurs at the lower end of the spectrum. The point target response are obtained with the sampling rate increased by a factor of 100.

The resulting point target response is shown in Fig. 8. The ionosphere induces a delay of 8.11 resolution cells in the target position, while the range spectral loss causes a relatively minor shift of -0.13 resolution cells in the target position. The simulated PPV and OPV are -0.166 and -0.067 rad, respectively, which are in close agreement with the approximated values of -0.169 and -0.067 rad computed using (24) and (27). The range spectral loss also leads to a broadening of the main lobe of the point target response.

In the absence of the ionosphere, the phase of the point target response is constant over time. After the ionospheric effect is introduced, the phase becomes time-varying. Notably, within the phase-stable region, the variation remains sufficiently small to be negligible. The range spectral loss leads to a linear phase ramp in the time domain, and the phase-stable region is broadened due to the reduced bandwidth.

Fig. 9 shows the variations in the peak position and the phase of the point target response with the bandwidth of range spectral loss at either end. The simulation results are in good agreement with the theoretical predictions. In particular, according to (31), both the PPV and OPV consist of two components: the change in the CSIP, which is induced primarily by the bandwidth change, and the SSIP, which is induced by the center frequency shift. The sum and difference of these two components from theoretical predictions show *excellent* agreement with the PPV and OPV obtained from simulations. The small perturbations observed in the simulated peak position shift and PPV are caused by signal discretization. In general, the peak position shift is relatively small, not exceeding one-third of a resolution cell. Within this range, the time-varying phase resulting from the ionosphere-induced FM-rate mismatch can be neglected.

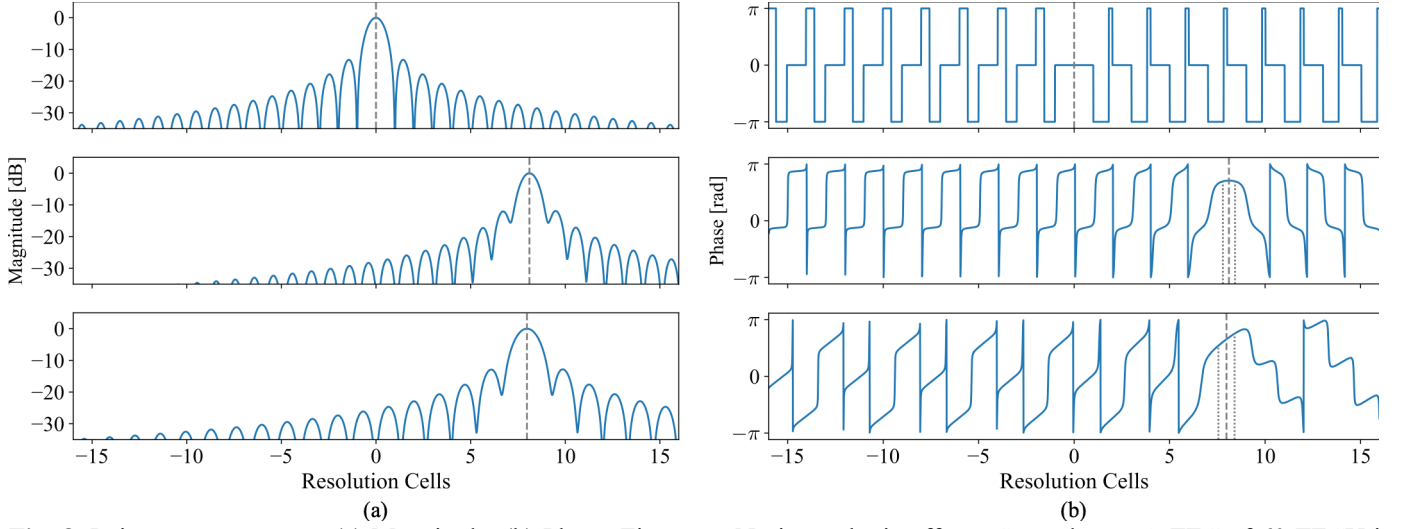


Fig. 8. Point target response. (a) Magnitude; (b) Phase. First row: No ionospheric effects. Second row: A TEC of 60 TECU is applied. Third row: A TEC of 60 TECU is applied together with a range spectral loss of $2\Delta f = 20$ MHz applied to the lower end of the spectrum. The gray dashed lines indicate the peak positions of the point target responses. The gray dotted lines indicate the phase-stable regions.

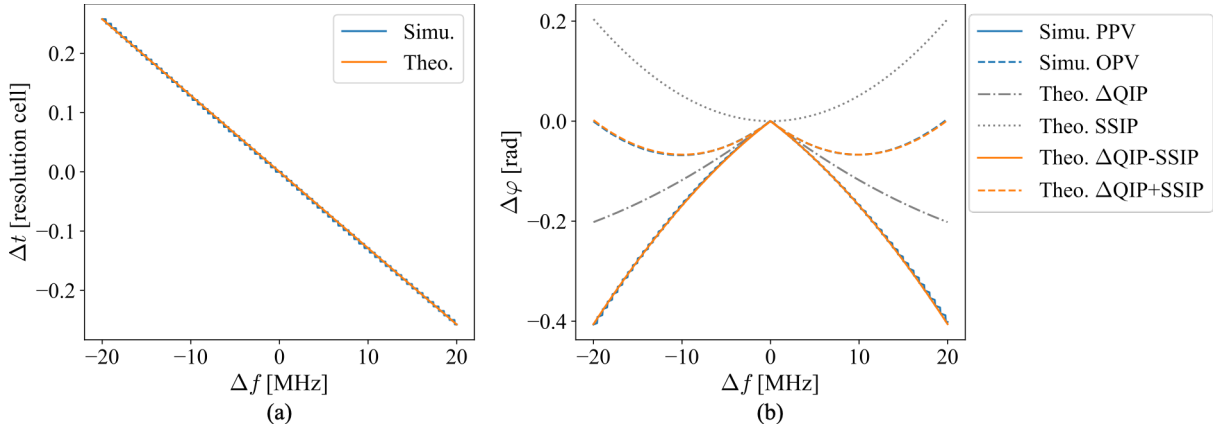


Fig. 9. Variations in (a) the peak position and (b) the phase of the point target response with the bandwidth of range spectral loss at either end. A TEC of 60 TECU is applied. The center frequency after range spectral loss is denoted by Δf , and the bandwidth of the lost spectrum is $2|\Delta f|$. If $\Delta f > 0$, the loss occurs at the lower end of the spectrum; if $\Delta f < 0$, it occurs at the higher end.

The range spectral loss caused by RFI removal does not necessarily occur exactly at the ends of the spectrum. Fig. 10 shows different spectral-loss scenarios and the corresponding imaging results. Compared to the first row, the peak position variations in the second, third, and fourth rows are 0, 0.01, and 0.03 resolution cells, respectively; the values of PPV are 0.085, 0.031, and -0.010 rad; the values of OPV are 0.085, 0.033, and -0.002 rad. Several key observations emerge from the simulations:

(1) Peak position: When spectral loss splits the spectrum into two or more segments, predicting the peak position of the superposed response is challenging. The intuitive rule that the peak shifts in the direction opposite to the loss location does not always hold. In general, however, a more asymmetric and edge-concentrated spectral loss induces a larger peak shift.

(2) Imaging quality: The main lobe narrows and the sidelobes intensify as range spectral loss shifts closer to the center of the spectrum. Conversely, the response magnitude deviate less from the standard sinc-function shape when range spectral loss occurs closer to the ends of the spectrum.

(3) Time-varying phase: The location of range spectral loss has a significant impact on the time-varying phase; however, a phase-stable region persists around the peak position. Within this region, the phase varies approximately linearly.

(4) PPV and OPV: Regardless of the location of range spectral loss, these phase variations remain minimal. From a relative perspective, the range spectral loss occurring at the spectral center exerts the most pronounced influence on these terms.

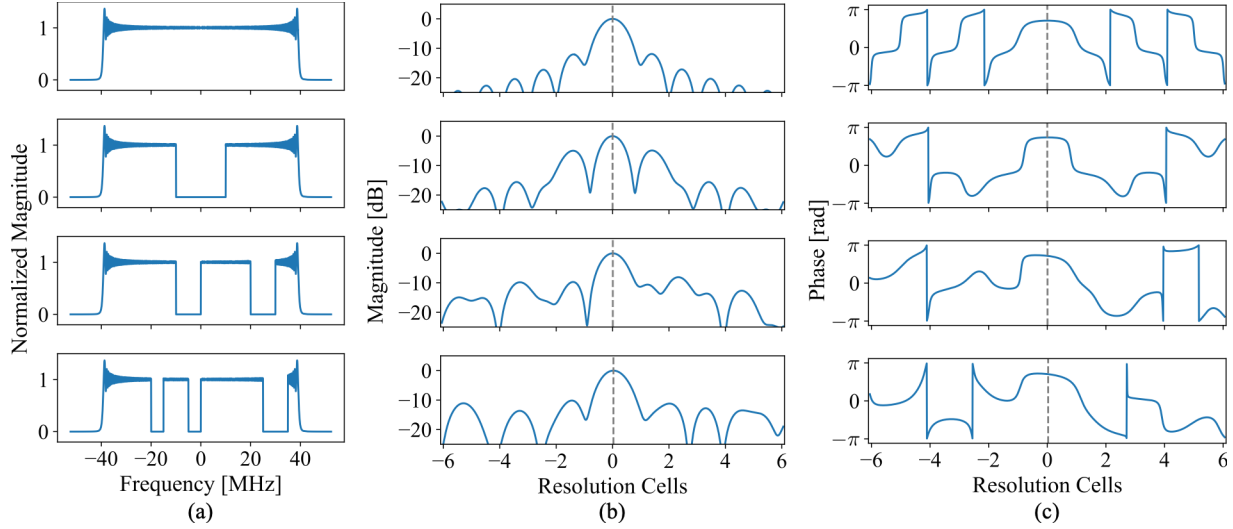


Fig. 10. (a) Range-spectral-loss scenarios and the corresponding (b) magnitude and (c) phase of the imaging results. The first row corresponds to the full-spectrum scenario, while for the other rows, a total range spectral loss of 20 MHz occurs.

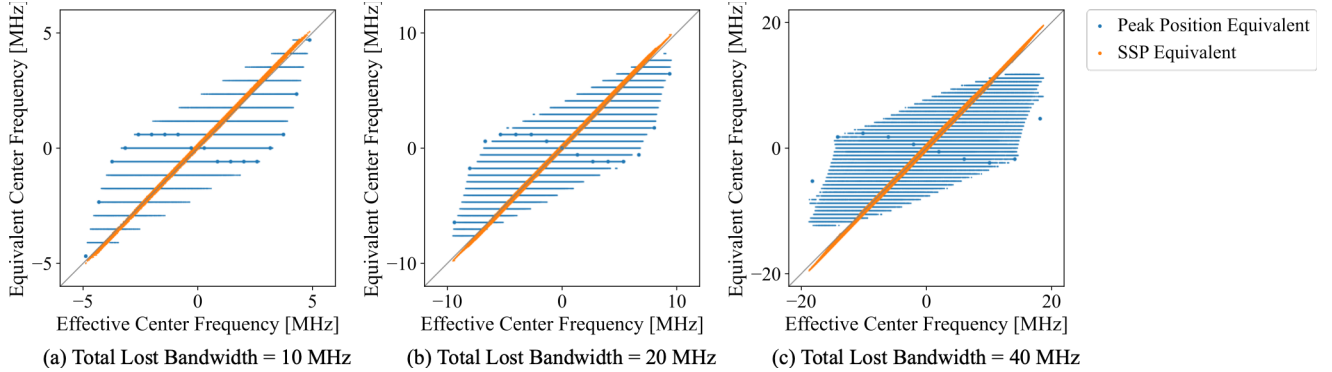


Fig. 11. Relationships between the effective center frequency and the peak-position equivalent center frequency, and between the effective center frequency and the SSP equivalent center frequency. The larger blue dots represent scenarios with a single lost spectral segment.

Fig. 11 examines the relationships between the effective center frequency and the peak-position equivalent center frequency, and between the effective center frequency and the SSP equivalent center frequency, all of which are defined in Section II-D. The simulations are performed for three total lost bandwidth values of 10, 20, and 40 MHz. To obtain a finite number of simulation scenarios, the continuous parameter space is discretized by considering up to three lost spectral segments. The bandwidth of each lost spectral segment is varied with an approximate interval of 1 MHz, while the bandwidth of each remaining spectral segment is varied with an approximate interval of 4 MHz. This discretization yielding 30,771, 79,815, and 90,685 scenarios for the three total lost bandwidths, respectively. The results show that the SSP equivalent center frequency is in excellent agreement with the effective center frequency. However, the peak-position equivalent center frequency exhibits varying degrees of deviations from the effective center frequency. The discontinuities in the figure are caused by insufficient oversampling of the signal.

B. Interferogram

To validate the theoretically derived interferometric phase under range spectral loss, we perform simulations for distributed targets. The TEC values in the reference and secondary signals are set to 80 TECU and 20 TECU, respectively. The offset between the reference and secondary signals can be precisely computed from the known imaging geometry and ionospheric delays, thereby allowing for precise coregistration. Furthermore, because the ionospheric effects are known, we can reconstruct the true target positions as they would be in the absence of the ionosphere, enabling precise removal of the geometric phase.

In each simulation, we repeat the entire process 500 times, with only the elementary scatterers on the ground being randomly generated. The results from the independent realizations are then averaged. Since the 500 pixels span a very small spatial extent in the high-resolution UBS mode, we can assume that Δf essentially remains constant across all pixels. Therefore, we further average the results over the range direction to obtain the final result.

Fig. 12 shows the phase of the interferogram between the reference signals with ($p_r(t)$) and without ($p'_r(t)$) range spectral loss. The target position $t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}$ of $p_r(t)$ is coregistered to the target position $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$ of $p'_r(t)$, i.e., coregistration at the actual position. The result is precisely predicted by (43). If both signals are coregistered at position $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$, i.e., coregistration at the interferometric effective position, the interferometric phase becomes zero. This validates that non-overlapping spectral components do not contribute to the expected interferometric phase. Although the original reference signal without range spectral loss retains its full spectrum, the non-overlapping spectral components are effectively “lost” during interferometric processing, resulting in the shift of the interferometric effective center frequency and the corresponding interferometric effective position.

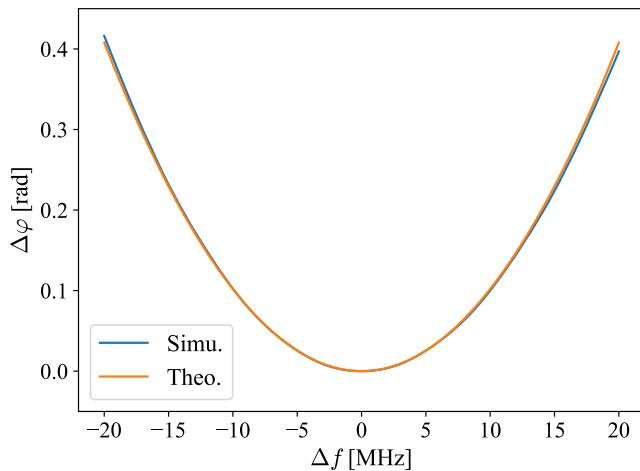


Fig. 12. Phase of the interferogram between the reference signals with and without range spectral loss, $p_r(t)$ and $p'_r(t)$. The interferogram is evaluated at $t_{\text{geom},r} + t_{\text{tropo},r} + \mu_{1,r}$ for $p_r(t)$, and $t_{\text{geom},r} + t_{\text{tropo},r} + \mu'_{1,r}$ for $p'_r(t)$, i.e., coregistered at the actual position.

Fig. 13 shows the interferometric phase between the reference and secondary signals, with the geometric phase and the DDIP removed. Two results are presented. As shown in Fig. 5, in the first scenario, the reference and secondary signals are coregistered at the actual position, whereas in the second scenario, they are coregistered at the interferometric effective position. The two results are well predicted by (48) and (49), respectively. Only minor discrepancies are observed. First, (45) shows that the frequencies of the reference and secondary signals are related by a linear transformation involving both translation and scaling. However, for simplicity, this transformation is approximated by an equivalent translation in the subsequent theoretical derivations. Second, the derivation of the interferometric phase in Appendices E and F involves several additional approximations. These factors might have contributed to the observed discrepancies.

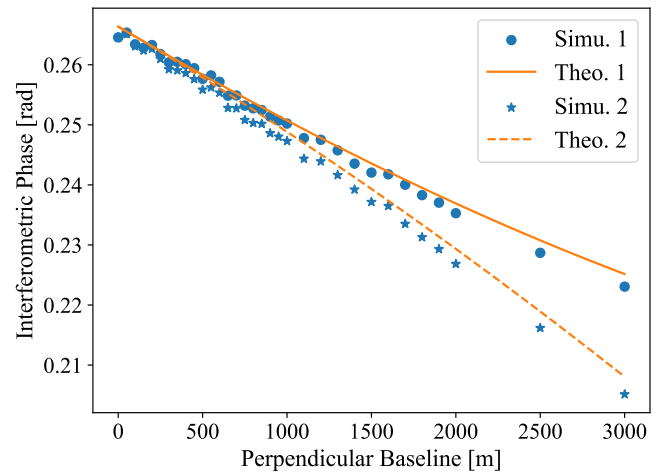


Fig. 13. Interferometric phase with the geometric phase and the DDIP removed. Scenario 1: The reference and secondary signals are coregistered at the actual positions corresponding to their full-band spectra. The resulting interferometric phase is computed using (48). Scenario 2: The reference and secondary signals are coregistered at the interferometric effective positions corresponding to their overlapping spectra. The resulting interferometric phase is computed using (49).

C. Range Split-Spectrum Method

We conduct simulations for the range split-spectrum method in the contexts of InSAR ionospheric phase estimation, absolute InSAR phase determination, and residual range offset estimation. As in Section V-B, unless otherwise stated, the TEC values in the reference and secondary signals are set to 80 TECU and 20 TECU, respectively.

We begin with InSAR ionospheric phase estimation using the trisecting scheme. The reference and secondary signals are bandpass filtered to extract the lower- and upper-band signals. The center frequency of each sub-band signal is shifted to zero after bandpass filtering. The sub-band signals are coregistered and then used to form the sub-band interferograms. Since the phase exhibits negligible variation along the range direction after removal of the geometric phase, phase unwrapping is not required. The 2π phase ambiguity is resolved using prior knowledge of the true DDIP.

Fig. 14 shows the sub-band interferometric phases, with the geometric phase and DDIP removed. Again, we present two results. In the first scenario, the sub-band signals are coregistered at positions corresponding to those of the full-band spectra. This strategy represents the approach commonly adopted in real data processing. In the second scenario, all four sub-band signals are coregistered to the position of the full-band reference signal. This represents an ideal scenario in which the target position variations induced by sub-band filtering are eliminated. The two results are well predicted by (54) and (48), respectively. As the sub-band bandwidth is reduced to one-third of the full bandwidth, DCSIP is reduced to one-ninth of its full-band value. Consequently, in the second scenario, the interferometric phase with the geometric phase and DDIP removed is much smaller than the full-band result shown in Fig. 13.

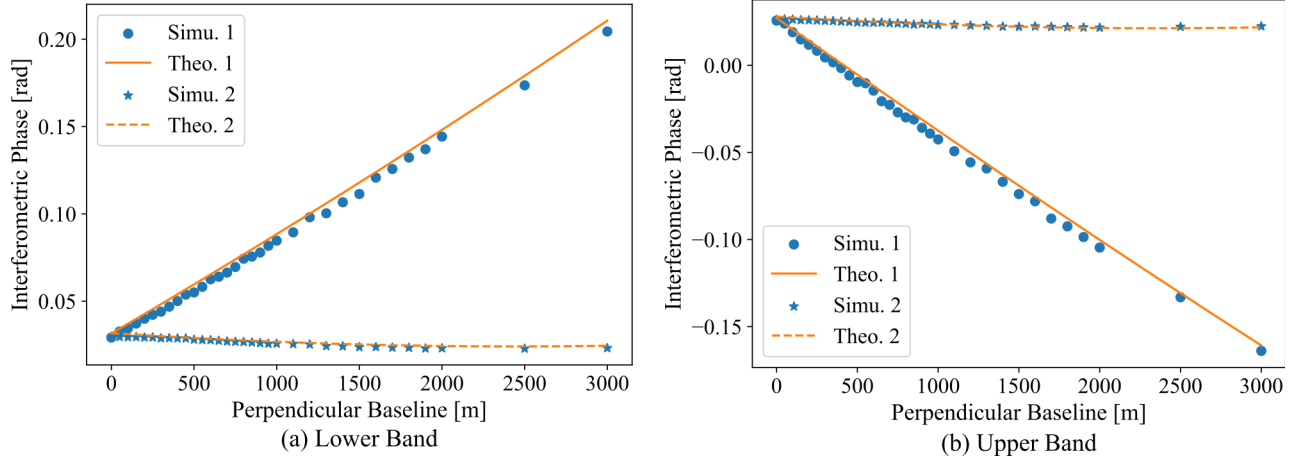


Fig. 14. The sub-band interferometric phase with the geometric phase and DDIP removed. Scenario 1: The sub-band signals are coregistered at positions corresponding to those of the full-band spectra. The resulting interferometric phase is computed using (54). Scenario 2: All four subband signals are coregistered to the position of the full-band reference signal. The resulting interferometric phase is computed using (48).

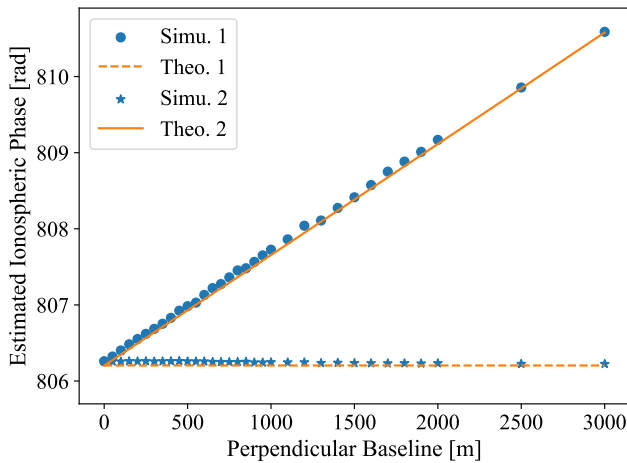


Fig. 15. Ionospheric phase estimated by the range split-spectrum method. Scenario 1: The sub-band signals are coregistered at positions corresponding to the full-band spectra. Scenario 2: All four subband signals are coregistered to the position of the full-band reference signal.

Fig. 15 shows the estimated ionospheric phase in the full-band interferogram for the two scenarios. In the second scenario, the orange dashed line represents the true DDIP, with the DCSIP and DSSIP terms in (48) omitted because of their negligible magnitudes. Adding the PVP given by (60) to the true DDIP yields the result shown by the orange solid line corresponding to the first scenario. These theoretical results agree well with the simulated results, thereby validating the derived PVP term. In the subsequent simulations of the range split-spectrum method, we only consider the first scenario and verify the corresponding error derived in Section IV.

For InSAR ionospheric phase estimation using the main-sideband scheme, the full-band signals are first filtered to extract the main band and sideband. The main-band signals are processed following the standard InSAR workflow, while

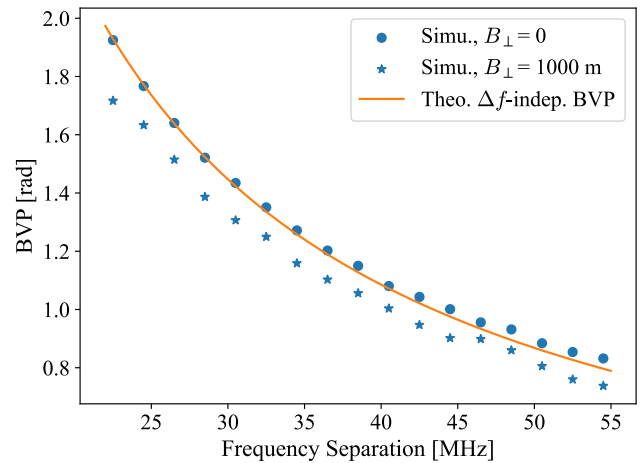


Fig. 16. BVP as a function of the separation between the center frequencies of the main band and the sideband. Theoretical Δf -independent component of BVP is computed using (64).

the sideband signals are coregistered using the offset derived from the main-band signals. Following geometric phase removal and phase ambiguity resolution, the main-band and sideband interferometric phases are used to estimate the InSAR ionospheric phase in the main-band interferogram. Consequently, the result contains both PVP and BVP.

Fig. 16 shows the BVP obtained by subtracting DDIP and PVP from the estimated ionospheric phase. Equation (64) accurately characterizes the BVP when the perpendicular baseline is zero. Although the BVP also depends on Δf , simulation results show that it varies only slightly even with a perpendicular baseline of up to 1000 m. Therefore, neglecting the dependence of BVP on Δf is justified, allowing (64) to serve as a reliable approximation.

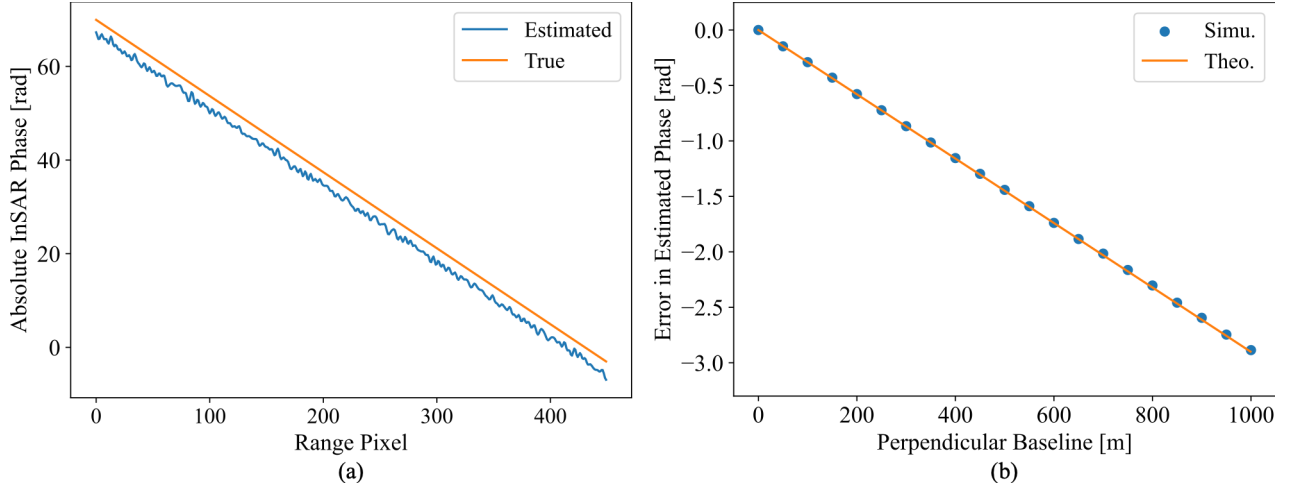


Fig. 17. Absolute InSAR phase determination by the range split-spectrum method. (a) A comparison of the estimated absolute phase and true absolute phase, which only contains geometric phase due to a perpendicular baseline of 1000 m. (b) Variation of the estimation error with the perpendicular baseline. The estimation error can be precisely computed using (67).

For absolute InSAR phase determination, we also adopt the trisecting scheme. The frequency separation is relatively large. Consequently, when the perpendicular baseline exceeds 1000 m, the difference between the interferometric phases of the two sub-bands falls outside the principal range of $(-\pi, \pi]$, rendering the method inapplicable. Accordingly, the perpendicular baseline is limited to no more than 1000 m in the simulation. The TEC values of the reference and secondary signals are set to be equal, both at 50 TECU. Therefore, the interferometric phase contains no ionospheric phase and consists solely of the geometric phase.

Fig. 17 (a) presents the absolute phase estimated using (66) with a perpendicular baseline of 1000 m. Compared with the true absolute phase, the estimation error is clearly observed. Fig. 17 (b) shows that the simulated estimation error increases with the perpendicular baseline, which is in good agreement with the theoretical estimation error predicted by (67).

Finally, we employ the trisecting scheme to estimate the residual range offset. Unlike the other two applications, the center frequency is not shifted to zero. We intentionally introduce an offset error of 0.2 pixels (approximately 0.15 resolution cells) during resampling. Two datasets from the previous experiment are employed: one with TEC values of 80 TECU and 20 TECU for the reference and secondary signals, respectively, and the other with a TEC value of 50 TECU for both signals.

Fig. 18 presents the estimated residual range offset. Again, the estimation error is clearly visible and increases with the perpendicular baseline. Since the two datasets have the same sum of the TEC values (i.e., $\sum \text{TEC}$), the range split-spectrum method is expected to yield identical estimates in theory, as expressed by (72). When $\Delta \text{TEC} = 0$, the simulation results agree well with the theoretical predictions. However, when $\Delta \text{TEC} \neq 0$, some minor discrepancies are observed. These can be attributed to various approximations made in the derivation of (72), which neglect certain terms related to ΔTEC .

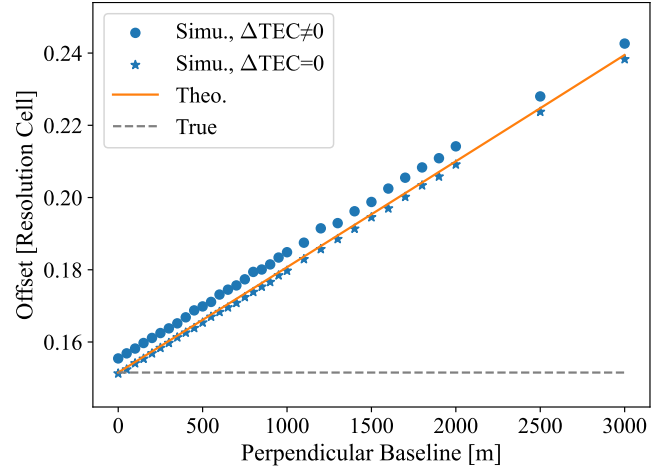


Fig. 18. Residual range offset estimation by the range split-spectrum method (known as the spectral-diversity method). The dots represent the simulation results with TEC values of 80 TECU and 20 TECU for the reference and secondary signals, respectively, while the stars correspond to the case where both signals have a TEC value of 50 TECU.

VI. REAL-DATA EXPERIMENT

We conduct several real-data experiments to validate the significant errors derived in Sections II-IV.

A. Point Target Response

We use high-resolution ALOS-2 UBS mode data containing corner reflectors [33] to analyze the point target response. The UBS mode is a stripmap mode. The data were acquired in the Kantō region of Japan. The corner reflector used in the analysis is shown in Fig. 19. We oversample the image by a factor of 32 and then apply bandpass filtering according to the trisecting scheme, without shifting the center frequency of each sub-band to zero.

Fig. 20 shows the range profiles of the oversampled response of the corner reflector. The peak positions of the

corner reflector in the upper and lower bands exhibit symmetric shifts of equal magnitude in opposite directions. The original peak position lies within the phase-stable region of the sub-bands. Within this region, the phase varies approximately linearly, with the phase slopes in the lower and upper bands having equal magnitudes and opposite signs.

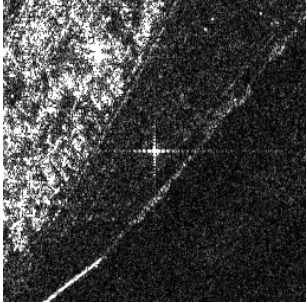


Fig. 19. A corner reflector in the SAR image acquired by ALOS-2 in UBS mode.

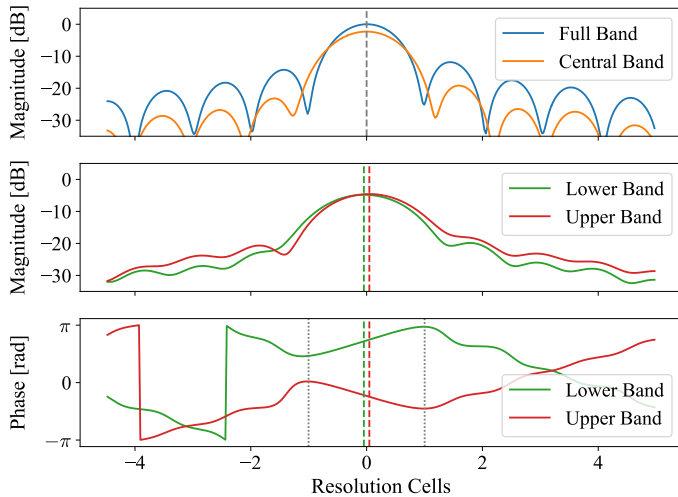


Fig. 20. Range profiles of the oversampled response of the corner reflector. The dashed lines indicate the peak positions, and the dotted lines denote the phase-stable region.

B. PVP in the Estimated InSAR Ionospheric Phase

We use wide-swath ALOS-2 WBD mode data to conduct experiments on the range split-spectrum method for InSAR ionospheric phase estimation. The WBD mode is a ScanSAR mode comprising five sub-swaths with a total swath width of 350 km, featuring an appreciable variation in Δf across the swath due to wavenumber shift. The data were acquired over the West Coast of the United States.

It is impossible to obtain the exact TEC experienced by the radar signals for validation. Fortunately, the ionospheric transfer function in (10) is linear in TEC. Therefore, we artificially introduce an additional TEC into the data and examine its effects. The additional TEC is approximately 65 TECU, corresponding to an exact 2-pixel shift (approximately

17.17 m) in the range direction. Using an integer-pixel shift avoids the need for resampling in the subsequent processing. We generate interferograms from the reprocessed data and estimate the resulting InSAR ionospheric phase using the ALOS-2 modules [34], [35] in the ISCE software [36]. Equation (59) shows that, by removing the InSAR ionospheric phase estimated using the original data, the phase components attributable solely to the artificially introduced TEC can be isolated. To further simplify the experiment, the same TEC is injected into both the reference and secondary images. Consequently, the ΔTEC remains unchanged, and so do the phase terms associated with it, including the DDIP, the DSSIP, and the DCSIP. In contrast, the ΣTEC increases, resulting in an increase in the PVP. Importantly, the PVP is the only phase error term that depends on ΣTEC and is also the dominant error term, since the DSSIP and DCSIP are generally negligible. Therefore, the difference between the ionospheric phases obtained from the reprocessed and original data is solely caused by the PVP arising from the additional TEC in the reprocessed data, which provides an excellent opportunity to validate the PVP. The PVP is a range-varying phase because Δf varies with range.

Fig. 21 shows the difference between the ionospheric phases obtained from the reprocessed and original data, attributable to the PVP arising from the additional TEC in the reprocessed data. The result is in good agreement with that predicted by (60). In the computation, a flat terrain is assumed, as the range split-spectrum method requires strong spatial filtering to estimate the ionospheric phase, which largely smooths out topography-related variations. The range-varying nature is clearly evident and, consequently, could obscure the measured long-wavelength tectonic motions.

The coupling of ΣTEC (or ΔTEC) and Δf in the phase error terms in (59) can give rise to a non-zero closure phase, which can be verified without artificially injecting TEC. To this end, we process a stack of three ScanSAR images (see Table IV for details). For each pair of images, we estimate the InSAR ionospheric phase using the standard workflow, but without filtering the estimate. Instead, the filtering is applied to the sum of the ionospheric phase estimates. The result in Fig. 22 clearly exhibits non-zero closure phases with a distinct range-varying pattern, which is primarily attributable to the PVP.

TABLE IV
SCAN SAR IMAGE STACK FOR VERIFYING THE CLOSURE
PHASE ERROR.

Image Index	Acquisition Date	Perpendicular Baseline [m]	TEC Level
1	2015/02/28	-344	High
2 (Reference)	2019/09/21	0	Low
3	2023/03/04	233	High

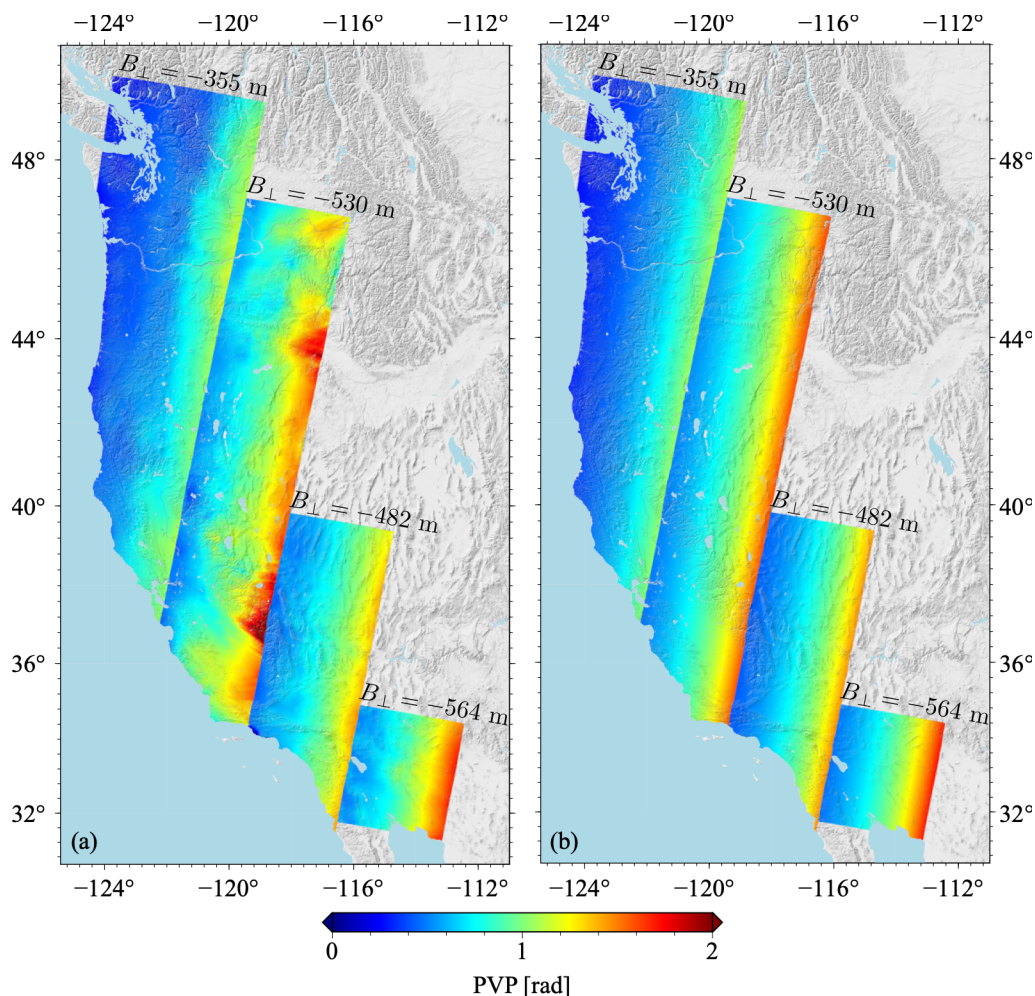


Fig. 21. Difference between the ionospheric phases from the reprocessed and original data, attributable to the PVP arising from the additional TEC in the reprocessed data. (a) Real-data result. (b) Theoretical result computed using (60). Region: The West Coast of the United States. Data: ALOS-2 WBD mode.

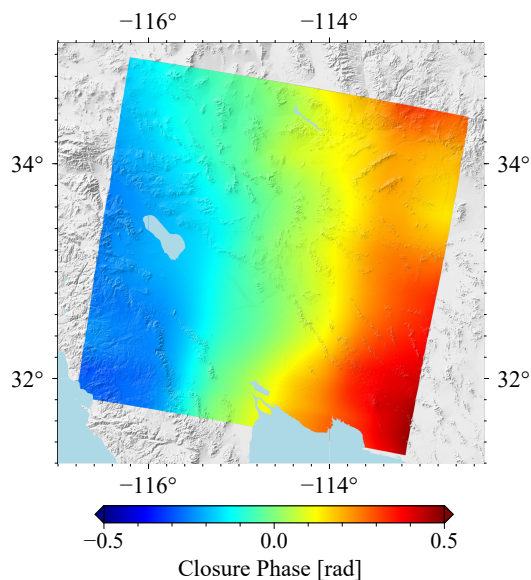


Fig. 22. Non-zero closure phase in the InSAR ionospheric phase estimated by the range split-spectrum method. Region: The West Coast of the United States. Data: ALOS-2 WBD mode.

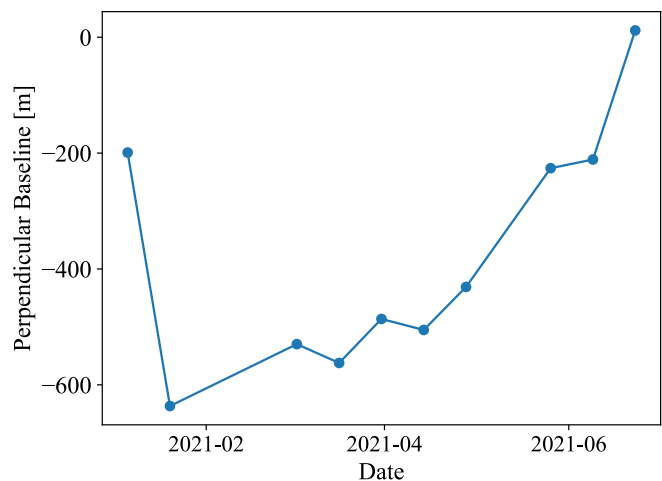


Fig. 23. Perpendicular baselines. Region: The West Coast of the United States. Data: ALOS-2 WBD mode.

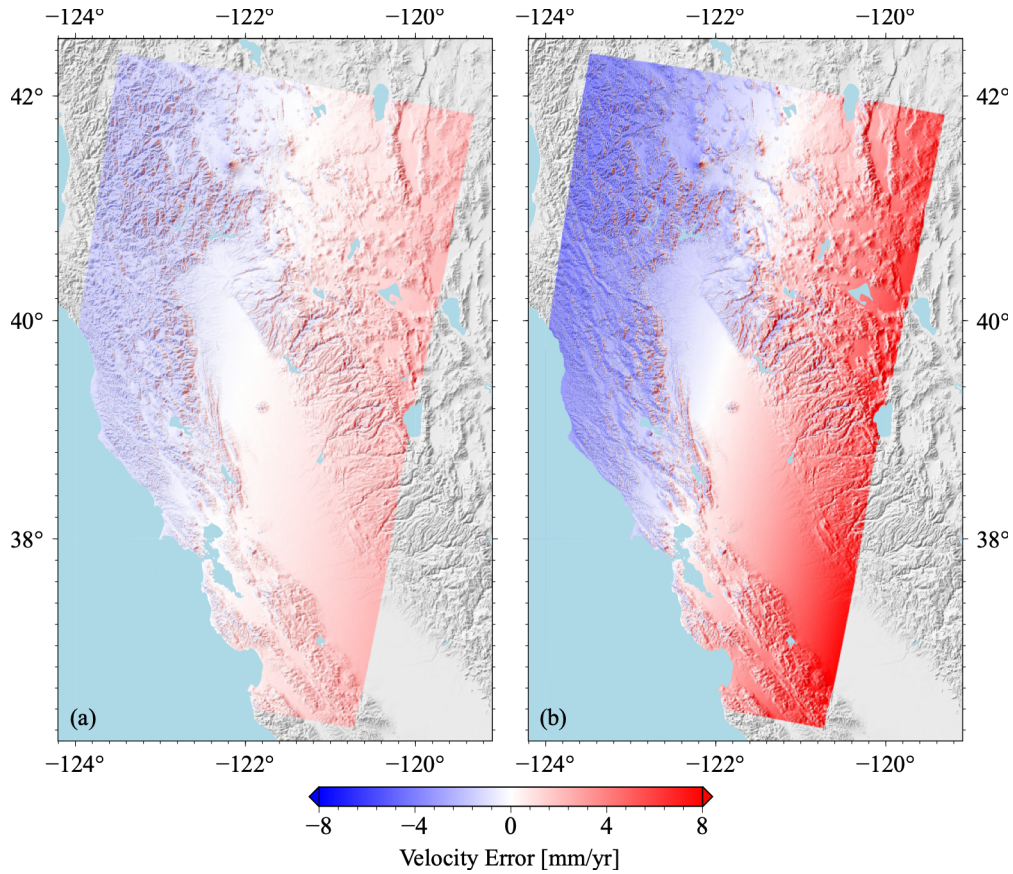


Fig. 24. Velocity error induced by the PVP. (a) Result from the stack of the first 9 acquisitions. (b) Result from the stack of the last 9 acquisitions.

As noted in Section IV-C and Appendix H, the PVP may introduce a range-varying bias in the InSAR time series, unless the perpendicular baseline is temporally uncorrelated. To verify this bias, we use 10 *consecutive* images. The perpendicular baselines shown in Fig. 23 clearly exhibit temporal correlation. Two stacks are processed: the first 9 acquisitions and the last 9 acquisitions. For each stack, we retrieve the TEC experienced by the SAR data using the JPLD Global Ionospheric Map (GIM) product [37]. We then compute the PVP according to (60). Finally, the PVP is allocated to each acquisition via least-squares adjustment performed in a fully connected network. The resulting PVP series are used to estimate a displacement velocity error.

The results are shown in Fig. 24. The temporal correlation of the perpendicular baselines in the second stack is substantially higher than that in the first stack. Consequently, the range ramp error in the velocity estimated from the second stack is significantly larger. The PVP here is computed using accurate values of Δf derived from the imaging geometry and the slope/aspect information extracted from a DEM. This result reflects the expected displacement velocity estimation error caused by the range split-spectrum method. This error exhibits not only an evident range-varying trend but also a strong topographic dependence, with distinct values for different slopes. However, as previously noted, such topographic details can be smoothed out in real data processing, which typically employs a strong spatial filter.

C. Error in the Estimated Residual Range Offset

We use ALOS-2 HBQ mode data acquired in stripmap mode to conduct experiments on the range split-spectrum method for residual range offset estimation. The data were acquired over the West Coast of the United States.

Similar to the estimation of the ionospheric phase, we artificially introduce an additional TEC into the data and examine its effects. The additional TEC is approximately 54 TECU, which corresponds to an exact 5-pixel shift (approximately 14.30 m) in the range direction. Each image is augmented by exactly the same TEC, resulting in an additional ionosphere-induced position delay that is identical across all images. The geometric offset is used to coregister both the original and reprocessed images. Consequently, the residual range offsets in the reprocessed data and the original data are also expected to be exactly the same. Hence, the observed difference between them is attributable solely to the estimation errors in the range split-spectrum method.

Fig. 25 shows the difference between the residual range offsets obtained from the two datasets. Note that although the noise level of the result is significant, it remains much lower than that of the ionospheric phase estimates. Therefore, no spatial filtering is applied after multilooking. In computing the theoretical error using (72), we have accurately computed Δf using a DEM instead of assuming flat terrain. The theoretical results agree well with the real data results, not only in the

range-varying trend but also in the distinct features on opposite sides of the slopes.

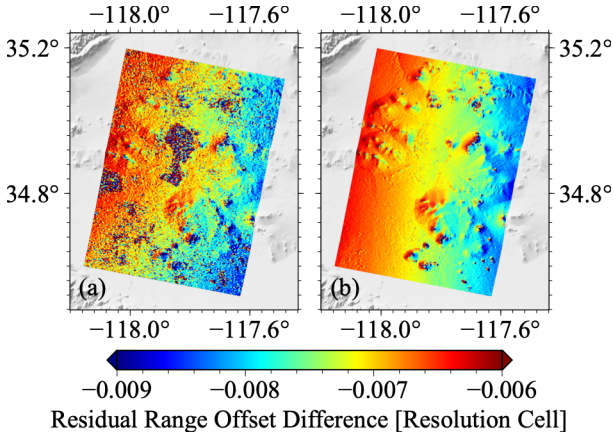


Fig. 25. Difference between the estimated residual range offsets from the reprocessed and original data. (a) Real-data result. (b) Theoretical result computed using (72). Region: The West Coast of the United States. Data: ALOS-2 HBQ mode. The range resolution cell size is 3.94 m, and the perpendicular baseline at the swath center of the reference image is 406 m.

VII. DISCUSSION

A. Correction of Range Spectral Loss Effects

The preceding analysis shows that the range spectral loss caused by the RFI generally has negligible effects on the point target peak phase. The range spectral loss induced by wavenumber shift also has negligible effects on the interferometric phase. The major concern lies in the application of the range split-spectrum method, where the combination of the two sub-band interferograms involves multiplication by a large coefficient, which significantly amplifies the errors. Under conditions of large perpendicular baselines or strong ionospheric activity, these amplified errors may become appreciable in practice.

We use ionospheric phase estimation as an illustrative example. Two types of errors are identified. The BVP, arising from the unequal bandwidths of the two bands, is inherent in the main-sideband scheme. It is proportional to ΔTEC . The BVP can be computed according to (64) by substituting the estimated ionospheric phase for the true DDIP. Since the BVP varies only weakly across the image and InSAR measures spatially relative phase changes, neglecting it is generally acceptable. The PVP, on the other hand, is caused by the coregistration misalignment induced by sub-band filtering and the shift in the interferometric effective center frequency due to wavenumber shift. It is proportional to $\sum \text{TEC} \cdot \Delta f$. In particular, Δf is pronounced in wide-swath modes, resulting in an appreciable range-dependent trend in the PVP.

Removing the PVP directly from the SAR data is challenging. Estimating the PVP-free ionospheric phase via a linear combination of sub-band interferograms requires the simultaneous cancellation of both the non-dispersive phase and the error term given by (57). This, in turn, requires at least

three interferograms from distinct bands. Under the trisecting scheme, the lower-, central-, and upper-band interferograms are available, and (58) can be reformulated as

$$\Delta\varphi_{\text{iono}} = \frac{f_u f_l (\Delta\varphi_l + \Delta\varphi_u - 2\Delta\varphi_c)}{2(f_c^2 - f_u f_l)} \quad (73)$$

where $\Delta\varphi_c$ is the interferometric phase of the central band. The issue, however, is that the coefficient multiplying the interferometric phases becomes excessively large, exceeding 2,000 for ALOS-2 UBS mode and approaching 100,000 for ALOS-2 ScanSAR mode. Such large coefficients drastically amplify the noise, rendering it impractical to obtain a reliable estimate via spatially filtering the ionospheric phase originally computed using (73).

Alternatively, the PVP can be computed using (60) with publicly available ionospheric VTEC products. However, these products are often provided in low spatial and temporal resolutions. Moreover, the radar signal does not traverse the entire ionospheric layer. These issues lead to large uncertainties in the TEC estimates [38] required by (60).

The randomness of perpendicular baselines effectively mitigates the influence of PVP in time series analysis, as shown in Appendix H. Provided that satellite orbit control remains stable, the PVP-induced effects are generally negligible.

B. Target-Type Dependence

The derivations concerning InSAR in Sections III and IV assume a distributed target. Now consider a target consisting of a single point scatterer, commonly referred to as a Permanent Scatterer (PS), located at the center of a resolution cell. While wavenumber shift still occurs, range spectral loss does not, i.e., $\Delta f = 0$. Consequently, the actual position and the interferometric effective position are identical, and the interferometric phases in (48) and (49) reduce to a unified form:

$$\Delta\varphi = \Delta\varphi_{\text{non-disp}} + \Delta\varphi_{\text{DIP}} + \Delta\varphi_{\text{CSIP}} \quad (74)$$

which is consistent with the point target response given in (16).

In a more general scenario, a resolution cell consists of a distributed target and several dominant point targets. Neglecting the cross terms arising from the complex conjugate multiplication between different targets, the interferometric result can be approximated as the superposition of the individual interferometric contributions from all targets. When accounting for the influence of wavenumber shift on the interferometric ionospheric phase, the results in (48) and (49) can be adapted by replacing Δf with $\beta\Delta f$, where β ($0 \leq \beta \leq 1$) denotes the weighting factor for the distributed target. β also depends on the combined effects of several other factors, including the center frequency, the local incidence angle and the distribution of dominant point targets. Therefore, when evaluating the PVP and other errors introduced by the range split-spectrum method, the weighting factor β may itself vary, further complicating the analysis. Nevertheless, ionospheric phase estimation using the range split-spectrum method relies on heavy multilooking and strong spatial filtering to obtain a

low resolution estimate with sufficient precision. Accordingly, the ground can be approximated as an ideal distributed target, i.e., $\beta = 1$.

C. Interplay of Multiple Range Spectral Loss Mechanisms

The range spectral loss due to the RFI affects both the interferometric phase and the quantities derived from the range split-spectrum method. The impact on the interferometric phase is twofold. First, the resulting reduction in effective bandwidth modifies the DCSIP. Second, the resulting spectral loss shifts the actual position, the interferometric effective position, and the effective center frequency. This gives rise to a phase effect analogous to the DSSIP, but an analytical formulation is not readily available.

Regarding the impact of the RFI on the range split-spectrum method, we take the PVP in ionospheric phase estimation as an illustrative example. The generation of the PVP is governed by two key factors: the nonzero effective center frequency during the interferometric process and the shift in the actual position caused by sub-band filtering. The first factor can be handled by the effective center frequency defined by (32), which applies to arbitrary RFI scenarios. The second factor can generally be neglected when the total lost bandwidth due to the RFI is small compared with the filter bandwidth.

Given a subband, assume that, due to the RFI, the interferometric effective center frequencies of the reference and secondary images are shifted from $-\Delta f$ and Δf to $-\Delta f + \delta f_r$ and $\Delta f + \delta f_s$, respectively. Accordingly, the phase error induced by coregistration misalignment in the subband interferometric phase, as given in (57), becomes

$$\begin{aligned} \varphi_{\text{mis}} &= 2\pi(-\Delta f + \delta f_r)\Delta\tau_r - 2\pi(\Delta f + \delta f_s)\Delta\tau_s \\ &= \frac{8\pi K_2(f_{\text{sub}} - f_c)}{cf_c^3} [\text{TEC}_r\delta f_r - \text{TEC}_s\delta f_s - \sum \text{TEC} \cdot \Delta f] \end{aligned} \quad (75)$$

The RFI-induced shifts in the interferometric effective center frequencies, δf_r and δf_s , may differ between the two subbands. Denoting the lower and upper sub-bands by the subscripts 'l' and 'u', the resulting error in the InSAR ionospheric phase estimated using (58) can be expressed as

$$\begin{aligned} \varphi_{\text{error}} &\approx \varphi_{\text{PVP}} - \frac{2\pi K_2}{cf_c^2} [\text{TEC}_r(\delta f_{l,r} + \delta f_{u,r}) \\ &\quad - \text{TEC}_s(\delta f_{l,s} + \delta f_{u,s})] \end{aligned} \quad (76)$$

It is worth noting that wavenumber shift must also be taken into account when computing δf_r and δf_s . According to Fig. 4, a fixed-frequency interference source may contaminate different portions of the ground spectrum in different observations, thereby further reducing the effective bandwidth available for interferometric processing and consequently exacerbating the impact of the RFI. In other words, if both the reference and secondary images contain an RFI spanning the frequency range $[f_1, f_2]$, then, when determining the interferometric effective center frequency, an additional lost spectral segment must be considered in addition to the spectral segments already identified as lost due to the wavenumber shift and RFI. For the reference image, this segment spans $[f_1 - 2\Delta f, f_2 - 2\Delta f]$, whereas for the secondary image, it spans $[f_1 + 2\Delta f, f_2 + 2\Delta f]$.

For the simplified case, where the perpendicular baseline is zero, $\text{PVP} = 0$, $\delta f_{l,r} = \delta f_{l,s}$, and $\delta f_{u,r} = \delta f_{u,s}$. The subscripts 'r' and 's' can be dropped, and (76) simplifies to

$$\varphi_{\text{error}} \approx -\frac{2\pi K_2 \Delta \text{TEC} (\delta f_l + \delta f_u)}{cf_c^2} \quad (77)$$

Equation (77) can be expressed as the DDIP multiplied by a coefficient of $-(\delta f_l + \delta f_u)/2f_c$. Assuming an RFI bandwidth of 2 MHz, this coefficient remains below 0.0005 for L-band. If the spatial variations of δf_l and δf_u are neglected, the impact of the RFI on ionospheric phase estimation can be considered negligible.

Building on the simulations in Section V, we conducted further experiments using the ALOS-2 UBS mode parameters. A 2 MHz range spectral loss due to the RFI was introduced into both the reference and secondary signals, and the InSAR ionospheric phase was then estimated using the range split-spectrum method. As shown in Fig. 26, the estimation results are in good agreement with those predicted by (76). When the perpendicular baseline is zero, the estimated ionospheric phase exhibits a piecewise linear dependence on the RFI center frequency, which is consistent with (77). As the baseline increases, however, the wavenumber shift effect expands the spectral range impacted by the RFI, leading to a more complex dependence.

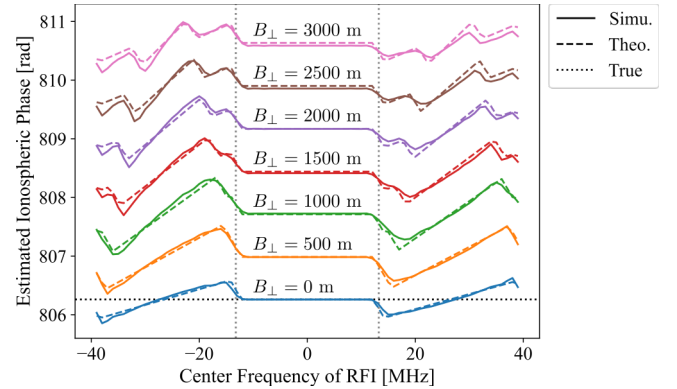


Fig. 26. Effect of the range spectral loss due to the RFI on the InSAR ionospheric phase estimated by the range split-spectrum method. The data contains both a 2 MHz range spectral loss due to the RFI and the range spectral loss due to the wavenumber shift as a function of perpendicular baseline, $B_{\perp}$. The vertical gray dotted lines mark the subband boundaries under the trisecting scheme.

VIII. CONCLUSION

Range spectral loss is common in InSAR and can arise from various mechanisms, including RFI removal, wavenumber shift, and intentional bandpass filtering. This study provides a unified analytical framework for understanding its effects on the point target response, the interferogram, and the estimates obtained via the range split-spectrum method. While detailed summaries of the theoretical results are provided in each of Sections II–IV, this section focuses on the overarching conclusions and their connections.

Spectral loss location distinguishes between two scenarios. In the simple scenario (e.g., wavenumber shift and intentional bandpass filtering), spectral loss occurs exclusively at the ends of the range spectrum, leaving a single continuous segment. The effects include a center-frequency shift and a bandwidth change. In the complex scenario (e.g., RFI), spectral loss occurs at arbitrary positions, and an effective center frequency can be defined.

Assuming the center frequency after spectral loss is not shifted to zero, the effects of range spectral loss can be categorized into two classes. The first class is independent of whether the system is dispersive or not. It appears as a center-frequency-dependent linear phase in the time domain, i.e., the SSP. In the complex scenario, the SSP is well predicted by the effective center frequency. The second class arises from dispersive processes, specifically, that associated with the ionosphere, which causes the peak position to depend on the center frequency and introduces a complex time-varying phase, the SIP. A phase-stable region exists around the peak, where the time-varying component VSIP can be neglected and only the constant component CSIP needs to be considered. In the simple scenario, after spectral loss, the peak position shifts with the center frequency, and the CSIP varies primarily with the bandwidth. In the complex scenario, however, no analytical solution is available.

The interferometric phase of a distributed target under various scenarios in this paper has been derived using two independent approaches: one based on coherent integration over elementary scatterers, and the other based on the point target model. The two approaches yield consistent results. Therefore, the effects of the simple loss scenario on InSAR can be characterized by the target position shift, the CSIP, and the SSP. The DCSIP component in the interferometric phase depends on the bandwidth after the loss. When the range split-spectrum method is used for ionospheric phase estimation, unequal subband bandwidths introduce the BVP error. The position shift and SSP are coupled: the position shift causes coregistration misalignment, while the SSP is a linear phase; their combination produces an additional phase term. In the interferometric phase, this manifests as the DSSIP, which depends on the coregistration scheme. In the ionospheric phase estimated by the range split-spectrum method, it appears as the PVP error. Similar errors are also present in other estimates via the range split-spectrum method.

At L-band, for a point target response, the CSIP is typically insignificant, generally not exceeding 0.3 rad. In the presence of range spectral loss, the resulting SSIP and the variation in the CSIP are even smaller, generally below 0.1 rad. Consequently, for the interferometric phase of a distributed target, the contributions from the DCSIP and DSSIP induced by wavenumber shift are negligible. Therefore, in the context of ionospheric phase correction, correcting for the DDIP alone is sufficient. However, when estimating the InSAR ionospheric phase using the range split-spectrum method, the PVP error is not negligible under conditions of high ionospheric activity and large perpendicular baselines. The

PVP can further give rise to non-zero closure phases among the estimated ionospheric phases, and substantially bias the deformation velocity estimates obtained from InSAR time-series analysis when the perpendicular baselines are temporally correlated. In the main-sideband scheme for InSAR ionospheric phase estimation, the BVP error is generally less than one-thousandth of the DDIP and can be neglected.

Both simulation and real-data experimental results further support our theoretical findings. Moreover, as discussed in Section VII, within our theoretical framework, preliminary analyses can also be conducted for more complex cases, including non-ideal distributed targets and scenarios involving multiple coexisting spectral loss mechanisms. Range spectral loss is a complex issue in InSAR, and our findings clarify various potential misinterpretations associated with its effects. In general, the effects on InSAR are negligible. However, the effects on applications employing the range split-spectrum method can be appreciable.

APPENDIX A

A SPECTRAL REPRESENTATION OF THE SAR RANGE SIGNAL FOR POINT TARGET

Assume a linear Frequency-Modulated (FM) signal transmitted by the radar:

$$s_{tx}(t) = \text{rect}\left\{\frac{t}{T_r}\right\} \cos\{2\pi f_c t + \pi K_r t^2\} \quad (\text{A.1})$$

where f_c is the carrier frequency, K_r is the linear FM rate, T_r is the signal duration, and t is the range time, whose origin is defined at the antenna phase center. The signal spectrum can be approximated via the Principle of Stationary Phase (POSP) [39]:

$$\begin{aligned} S_{tx}(f) &= \mathcal{F}\{s_{tx}(t)\} \\ &= \mathcal{F}\left\{\frac{1}{2}\text{rect}\left\{\frac{t}{T_r}\right\} \exp\{j2\pi f_c t + j\pi K_r t^2\} \right. \\ &\quad \left. + \frac{1}{2}\text{rect}\left\{\frac{t}{T_r}\right\} \exp\{-j2\pi f_c t - j\pi K_r t^2\}\right\} \\ &\approx \frac{1}{2}\text{rect}\left\{\frac{f+f_c}{|K_r|T_r}\right\} \exp\left\{j\pi \frac{(f+f_c)^2}{K_r}\right\} \\ &\quad + \frac{1}{2}\text{rect}\left\{\frac{f-f_c}{|K_r|T_r}\right\} \exp\left\{-j\pi \frac{(f-f_c)^2}{K_r}\right\} \\ &= S_{tx,n}(f) + S_{tx,p}(f) \end{aligned} \quad (\text{A.2})$$

where f is the range frequency. Here, the constant term introduced by the POSP has been omitted for simplicity. Accordingly, its spectral range comprises a negative frequency component and a positive frequency component that are symmetric about zero:

$$\left[-f_c - \frac{|K_r|T_r}{2}, -f_c + \frac{|K_r|T_r}{2}\right] \cup \left[f_c - \frac{|K_r|T_r}{2}, f_c + \frac{|K_r|T_r}{2}\right] \quad (\text{A.3})$$

Propagation of the signal involves a number of physical processes. The result of each process can be viewed as the response of the respective system to the signal. As such, the signal can be considered as passing through the target backscattering system, the geometric system, the tropospheric system, and the ionospheric system, each of which can be reasonably approximated as a Linear Time-Invariant (LTI)

system during the short period of signal propagation. Therefore, the spectrum of the received signal $s_{rx}(t)$ can be represented as

$$S_{rx}(f) = S_{tx}(f) \cdot H_{bs}(f) \cdot H_{geom}(f) \cdot H_{tropo}(f) \cdot H_{iono}(f) \quad (A.4)$$

where $H_{bs}(f)$, $H_{geom}(f)$, $H_{tropo}(f)$, and $H_{iono}(f)$ are the transfer functions of these systems, respectively.

Quadrature demodulation shifts the positive frequency component of the spectrum to baseband by multiplying the received signal by a linear phase term and discarding the negative frequency component via lowpass filtering, which can be mathematically expressed as

$$\begin{aligned} s_{demod}(t) &= \text{LPF}\{s_{rx}(t) \cos 2\pi f_c t\} + j \\ &\quad \cdot \text{LPF}\{-s_{rx}(t) \sin 2\pi f_c t\} \\ &= \text{LPF}\{s_{rx}(t) \cdot (\cos 2\pi f_c t - j \sin 2\pi f_c t)\} \\ &= \text{LPF}\{s_{rx}(t) \cdot \exp\{-j2\pi f_c t\}\} \end{aligned} \quad (A.5)$$

where LPF denotes the lowpass filtering operator. The resulting signal spectrum is given by

$$S_{demod}(f) = S_{tx,p}(f + f_c) \cdot H_{bs}(f + f_c) \cdot H_{geom}(f + f_c) \cdot H_{tropo}(f + f_c) \cdot H_{iono}(f + f_c) \quad (A.6)$$

where

$$S_{tx,p}(f + f_c) = \frac{1}{2} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} \exp\left\{-j\pi \frac{f^2}{K_r}\right\} \quad (A.7)$$

Subsequent matched filtering further removes the linear FM component of the signal, yielding the focused signal:

$$S_{focus}(f) = \frac{1}{2} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} \cdot H_{bs}(f + f_c) \cdot H_{geom}(f + f_c) \cdot H_{tropo}(f + f_c) \cdot H_{iono}(f + f_c) \quad (A.8)$$

In summary, quadrature demodulation and matched filtering jointly eliminate all time-varying characteristics of the signal phase or, essentially, perform a deconvolution process, leaving only a phasor representation of the signal component arising from the physical processes in propagation. This signal component includes transfer functions $H_{bs}(f + f_c)$, $H_{geom}(f + f_c)$, $H_{tropo}(f + f_c)$, and $H_{iono}(f + f_c)$. The rectangular function, which corresponds to a sinc function in the time domain, defines the baseband spectral range. The $+f_c$ term in each spectral component arises from downconverting the spectrum to baseband during quadrature demodulation. From the perspective of the target, its reflectivity $H_{bs}(f + f_c)$ passes through the SAR system characterized by its range bandwidth, the geometric system $H_{geom}(f + f_c)$, the tropospheric system $H_{tropo}(f + f_c)$, and the ionospheric system $H_{iono}(f + f_c)$, ultimately becoming the obtained signal $S_{focus}(f)$.

APPENDIX B

EVALUATION OF THE RESIDUAL ERRORS FOLLOWING IONOSPHERIC PHASE APPROXIMATIONS

Here, we evaluate the errors arising from the approximations in (9). In the first approximation, the higher-order (cubic and quartic) terms of $1/f$ are neglected. According to [40], using typical values for the Earth's magnetic flux density and other parameters, the ratio of the neglected terms to the retained term can be approximated as

$$r_1 \approx \frac{2.8 \times 10^5}{f} \left[\pm 1 + \frac{7.2 \times 10^7 + 2.1 \times 10^5}{f} \left(1 + \frac{5.6 \times 10^5 + 2.3 \times 10^3}{f} \right) \right] \quad (B.1)$$

The resulting ratio r_1 is substantially smaller than unity. For example, considering the carrier frequency of the ALOS-2 UBS mode, $f = 1.258$ GHz, we obtain $|r_1| < 0.000236$, which is negligible.

The second approximation stems from a Taylor series expansion, which can be formulated as a geometric series with a common ratio $q = -(f - f_c)/f_c$. Therefore, the relative contribution of the neglected third- and higher-order terms is given by

$$r_2 = |q^3| = \left| -\frac{(f - f_c)^3}{f_c^3} \right| \leq \frac{B^3}{8f_c^3} \quad (B.2)$$

where B is the signal bandwidth. This contribution is likewise well below unity. For example, for the ALOS-2 UBS mode with a bandwidth of $B = 79.4$ MHz, $|r_2| < 0.000033$. Reference [41] indicates that for wideband SAR satellites operating in the VHF/UHF bands, the cubic term must be considered. For commonly used frequency bands (P, L, C, etc.), however, retaining up to the quadratic term is generally sufficient.

At L-band, the ionospheric phase typically ranges from tens to hundreds of radians. Given the small magnitude of $|r_1|$ and $|r_2|$, the absolute error introduced by these two approximations generally does not exceed 0.1 rad. Consequently, the errors are negligible in both absolute and relative terms. Moreover, we observe that $|r_1|$ is nearly an order of magnitude larger than $|r_2|$. Therefore, once the first approximation is accepted, as is common practice, retaining higher-order terms in the Taylor series expansion offers no practical benefit, and the expansion only needs to be carried out up to the second-order term. This conclusion applies to all the satellites and imaging modes listed in Table B.I.

TABLE B.I
PARAMETERS OF REPRESENTATIVE SATELLITES AND IMAGING MODES.

Satellite	Frequency Band	Imaging Mode	Center Frequency [GHz]	Bandwidth [MHz]
BIOMASS	P	Stripmap	0.435	6
ALOS-2	L	Stripmap (UBS)	1.258	79.4
		Stripmap (HBQ)	1.236	38.0
		Stripmap (FBD)	1.236	24.9
		ScanSAR (WBD)	1.236	11.9

Satellite	Frequency Band	Imaging Mode	Center Frequency [GHz]	Bandwidth [MHz]
ALOS-4	L	Stripmap (UWD)	1.258	82.0
		Stripmap (HBQ)	1.237	40.0
		Stripmap (FWD)	1.237	27.0
LuTan-1	L	Stripmap	1.260	80.0
NISAR	L	SweepSAR (20+5)	1.229 (1.294)	20.0 (5.0)
		SweepSAR (40+5)	1.239 (1.294)	40.0 (5.0)
		SweepSAR	1.258	77.0
Sentinel-1	C	SweepSAR	3.2	75
		TOPS (IW1/IW2/IW3)	5.405	56.5/48.3/42.8

APPENDIX C

INVERSE FOURIER TRANSFORM OF THE QUADRATIC PHASE TERM

Here, we focus solely on the rectangular function representing the frequency range and the quadratic phase term in (10) and derive the corresponding time-domain result via the inverse Fourier transform:

$$s_{\text{qdt}}(t) = \int_{-\infty}^{\infty} \text{rect}\left\{\frac{f}{|K_r|T_r}\right\} \exp\{j\pi\mu_2 f^2\} \exp\{j2\pi f t\} df \quad (\text{C.1})$$

$$= \int_{-\frac{B}{2}}^{\frac{B}{2}} \exp\{j\pi\mu_2 f^2\} \exp\{j2\pi f t\} df$$

Because μ_2 is extremely small, the time-bandwidth product of the signal is also small; consequently, the Principle Of Stationary Phase (POSP) is not applicable. Although an analytical solution for this integral can be derived using Fresnel integrals, the result lacks intuitive clarity. Instead, we approximate the integral by expanding $\exp\{j\pi\mu_2 f^2\}$ to the first order in a Taylor series about $f = 0$, yielding

$$s_{\text{qdt}}(t) \approx \int_{-\frac{B}{2}}^{\frac{B}{2}} (1 + j\pi\mu_2 f^2) \exp\{j2\pi f t\} df \quad (\text{C.2})$$

$$= \int_{-\frac{B}{2}}^{\frac{B}{2}} \exp\{j2\pi f t\} df + j\pi\mu_2 \int_{-\frac{B}{2}}^{\frac{B}{2}} f^2 \exp\{j2\pi f t\} df$$

Consequently, $s_{\text{qdt}}(t)$ is decomposed into two components. The first component, which is independent of the ionosphere, is the inverse Fourier transform of the rectangular function:

$$s_{\text{qdt},1}(t) = \int_{-\frac{B}{2}}^{\frac{B}{2}} \exp\{j2\pi f t\} df = \frac{\sin \pi B t}{\pi t} = B \text{sinc}\{B t\} \quad (\text{C.3})$$

The second component is caused by the ionosphere-induced quadratic phase and is mathematically related to the second derivative of the first component:

$$s_{\text{qdt},2}(t) = j\pi\mu_2 \int_{-\frac{B}{2}}^{\frac{B}{2}} f^2 \exp\{j2\pi f t\} df \quad (\text{C.4})$$

$$= -\frac{j\mu_2}{4\pi} \frac{d^2 s_{\text{qdt},1}(t)}{dt^2}$$

$$= \frac{j\mu_2}{4\pi} \left(\frac{\pi B^2 \sin \pi B t}{t} + \frac{2B \cos \pi B t}{t^2} - \frac{2 \sin \pi B t}{\pi t^3} \right)$$

Therefore, $s_{\text{qdt}}(t)$ can be expressed as

$$s_{\text{qdt}}(t) \approx s_{\text{qdt},1}(t) + s_{\text{qdt},2}(t)$$

$$= \frac{\sin \pi B t}{\pi t} + \frac{j\mu_2}{4\pi} \left(\frac{\pi B^2 \sin \pi B t}{t} + \frac{2B \cos \pi B t}{t^2} - \frac{2 \sin \pi B t}{\pi t^3} \right) \quad (\text{C.5})$$

Its phase is given by

$$\arg\{s_{\text{qdt}}(t)\} \approx \arctan \left\{ \frac{\pi\mu_2 B^2}{4} \left[1 + \frac{2 \cos(\pi B t)}{\pi B t \sin(\pi B t)} - \frac{2}{\pi^2 B^2 t^2} \right] \right\} \quad (\text{C.6})$$

$$= \arctan \left\{ \frac{\pi\mu_2 B^2}{4} [1 + 2g(\pi B t)] \right\}$$

where $g(x)$ is defined as

$$g(x) = \frac{x \cos x - \sin x}{x^2 \sin x} = \frac{-\frac{1}{3}x^3 + \frac{1}{30}x^5 - \frac{1}{840}x^7 + \dots}{x^3 - \frac{1}{6}x^5 + \frac{1}{120}x^7 - \dots} \quad (\text{C.7})$$

For the approximation in (C.2) to remain valid, the condition $|\pi\mu_2 f^2| \ll 1$ should hold throughout the integration range. Using the parameters from the ALOS-2 UBS mode ($f_c = 1.258$ GHz and $B = 79.4$ MHz) in Table B.I and assuming $\text{TEC} = 60$ TECU, we obtain

$$|\pi\mu_2 f^2| \leq \left| \pi\mu_2 \left(\frac{B}{2} \right)^2 \right| = \frac{\pi K_2 B^2}{c f_c^3} \text{TEC} \approx 0.81 \quad (\text{C.8})$$

Although $|\pi\mu_2 f^2|$ approaches 1 at the integration boundaries, Fig. C.1 shows that, within a resolution cell around the peak position, (C.6) still provides an accurate approximation of the phase of $s_{\text{qdt}}(t)$ in (C.1). Given that the bandwidth at L-band is typically less than 80 MHz, the approximation in (C.2) is sufficiently accurate.

Following the convention in (6), we decompose the phase in (C.6), termed the Secondary Ionospheric Phase (SIP), into a Constant Secondary Ionospheric Phase (CSIP), φ_{CSIP} , and a time-Varying Secondary Ionospheric Phase (VSIP), φ_{VSIP} . The CSIP is defined at the peak position $t = 0$, which is equal to one-third of the Quadratic Phase Error (QPE) [39]. Accordingly, the time for the VSIP is measured relative to the peak position. For $|x| \leq 1$, $g(x) \approx -1/3$. Since $Bt = x/\pi$, the VSIP is approximately zero within a region of $1/\pi$ resolution cell centered around the peak position; this region is termed the phase-stable region. Depending on accuracy requirements, the radius of the phase-stable region can be extended up to $1/3$ or even $1/2$ resolution cell.

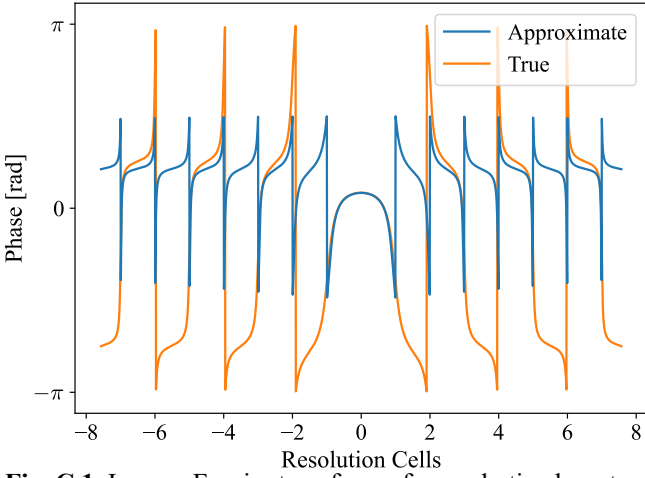


Fig. C.1. Inverse Fourier transform of a quadratic phase term.

APPENDIX D

UNDERLYING MECHANISM OF THE SSP

Assume that the transfer function of an LTI system is $H(f) = |H(f)| \exp\{j\phi(f)\}$. When a narrowband signal passes through this system [42], the phase delay in units of time is given by

$$\tau_p(f) = -\frac{\phi(f)}{2\pi f} \quad (\text{D.1})$$

and the group delay in units of time is given by

$$\tau_g(f) = -\frac{1}{2\pi} \frac{d\phi(f)}{df} \quad (\text{D.2})$$

For SAR signals, the range bandwidth B is typically much smaller than the carrier frequency f_c ; therefore, the signal can be considered narrowband. When considering the influence of the ionosphere on SAR signals, according to (10), $\phi(f) = \mu_0 - 2\pi\mu_1(f - f_c) + \pi\mu_2(f - f_c)^2$. In this case, the phase delay in radians is given by $2\pi f_c \cdot \tau_p(f_c) = -\mu_0$, and the group delay in units of time is $\tau_g(f_c) = \mu_1$. These results are consistent with the phase advance and position delay derived in Section II-B.

When range spectral loss occurs as illustrated in Fig. 1, the variation in phase delay due to the shift in center frequency is given by

$$\begin{aligned} \Delta\phi_p &= 2\pi(f_c + \Delta f) \cdot \tau_p(f_c + \Delta f) - 2\pi f_c \cdot \tau_p(f_c) \\ &= -\phi(f_c + \Delta f) + \phi(f_c) \end{aligned} \quad (\text{D.3})$$

According to the Lagrange mean-value theorem, there exists a frequency ξ satisfying $f_c < \xi < f_c + \Delta f$, such that

$$\Delta\phi_p = -\Delta f \left. \frac{d\phi(f)}{df} \right|_{f=\xi} \quad (\text{D.4})$$

On the other hand, a center-frequency shift from 0 to Δf corresponds to a linear phase $2\pi\Delta f t$ in the time domain. When this linear phase exactly cancels the aforementioned change in phase delay, i.e., $-\Delta\phi_p + 2\pi\Delta f t = 0$, we obtain

$$t = -\frac{1}{2\pi} \left. \frac{d\phi(f)}{df} \right|_{f=\xi} = \tau_g(\xi) \quad (\text{D.5})$$

In other words, following the occurrence of range spectral loss, there exists a specific temporal position in the time-domain point target response where the phase is identical to

that of the original point target response. This implies that range spectral loss has no effect on the phase at this particular position, i.e., there exists a phase-invariant point. Therefore, the total effect of the center-frequency shift on the phase, i.e., the spectral-shifting phase, is $2\pi\Delta f \Delta t$, where Δt denotes the time offset relative to the phase-invariant point $\tau_g(\xi)$.

For a non-dispersive system, the group delay $\tau_g(f)$ is independent of frequency, so $\tau_g(f_c) = \tau_g(\xi) = \tau_g(f_c + \Delta f)$. Hence, the peak position remains unchanged following range spectral loss, and the phase at that position is also unchanged. For a dispersive system such as the ionosphere, $\tau_g(f)$ depends on frequency. Consequently, the peak position shifts after range spectral loss. If we adopt the approximated transfer function in (10), we have $\xi = f_c + \Delta f/2$. In this case, the position corresponding to $\tau_g(\xi)$ can be approximated as the midpoint of the peak positions before and after range spectral loss, which explains why the SSP terms contained in the PPV and OPV are approximately equal in magnitude but opposite in sign.

It should be noted that this discussion assumes that the phase of the point target response of the full spectrum is constant, i.e., the time-varying phase is zero. According to Section II, a non-dispersive system meets this requirement. For a dispersive system such as the ionosphere, a phase-stable region exists near the peak position, and within this region, the requirement is also satisfied.

APPENDIX E

SSP FOR ARBITRARY RANGE SPECTRAL LOSS

The general case of multiple, arbitrarily located lost spectral segments, as shown in Fig. 2, can be approached via coherently summing the individual point target responses from each segment of the remaining spectrum. As discussed in Section II-C and Appendix C, each segment yields a signal with a slightly shifted peak position and a slightly altered peak phase relative to the full-spectrum response. Within the phase-stable region around each peak position, the phase associated with each segment varies approximately linearly due to the SSP. In the absence of the ionosphere, the amplitude associated with each segment exhibits a sinc-shaped pattern, with its peak amplitude depending on the bandwidth of the segment, according to (C.3).

For the superposed response, although it's challenging to derive a closed-form analytical expression for the resulting peak position, t_0 , we focus on the local region around the resulting peak position. Within this region, we denote the contribution from the i -th segment as $s_i(t)$, and make the following simplifying assumptions:

- (1) All $s_i(t)$ have the same phase φ_0 at the resulting peak position, and
- (2) The amplitude of each $s_i(t)$ remains constant within this region.

Each $s_i(t)$ also contains a linear phase term, corresponding to the SSP discussed in Appendix D. Then, $s_i(t)$ can be

expressed as

$$s_i(t) \approx B_i \exp\{j\varphi_0\} \exp\{j2\pi\Delta f_i(t - t_0)\} \quad (\text{E.1})$$

where B_i and Δf_i are the bandwidth and center frequency of the i -th spectral segment, respectively, and n is the total number of remaining spectral segments.

The superposed signal is given by

$$s_0(t) = \sum_{i=1}^n s_i(t) = \exp\{j\varphi_0\} \sum_{i=1}^n B_i \exp\{j2\pi\Delta f_i(t - t_0)\} \quad (\text{E.2})$$

The simulation results in Section V-A show that the phase variation of $s_0(t)$ near t_0 is approximately linear. The instantaneous frequency of $s_0(t)$ at $t = t_0$ is given by

$$\begin{aligned} f_{\text{inst}}(t_0) &= \frac{1}{2\pi} \frac{d}{dt} \arg\{s_0(t)\} \Big|_{t=t_0} \\ &= \frac{1}{2\pi} \text{Im} \left\{ \frac{s'_0(t)}{s_0(t)} \right\} \Big|_{t=t_0} \\ &= \frac{1}{2\pi} \text{Im} \left\{ \frac{\sum_{i=1}^n j2\pi B_i \Delta f_i \exp\{j2\pi\Delta f_i(t - t_0)\}}{\sum_{i=1}^n B_i \exp\{j2\pi\Delta f_i(t - t_0)\}} \right\} \Big|_{t=t_0} \\ &= \frac{\sum_{i=1}^n B_i \Delta f_i}{\sum_{i=1}^n B_i} \end{aligned} \quad (\text{E.3})$$

This result can be interpreted as the equivalent center frequency corresponding to the SSP of the superposed signal after spectral loss.

Alternatively, $s_0(t)$ can be viewed as the full-spectrum result minus the contributions from the lost spectral segments due to, e.g., the RFI. Under the above assumptions, $s_0(t)$ can be expressed as

$$s_0(t) = \exp\{j\varphi_0\} \cdot \left[B - \sum_{i=1}^m B_{\text{RFI}_i} \exp\{j2\pi\Delta f_{\text{RFI}_i}(t - t_0)\} \right] \quad (\text{E.4})$$

where B is the bandwidth of the full spectrum, B_{RFI_i} and Δf_{RFI_i} are the bandwidth and center frequency of the i -th RFI, respectively, and m is the total number of RFIs. Following a similar derivation, (E.3) can be rewritten as

$$f_{\text{inst}}(t_0) = -\frac{\sum_{i=1}^m B_{\text{RFI}_i} \Delta f_{\text{RFI}_i}}{B - \sum_{i=1}^m B_{\text{RFI}_i}} \quad (\text{E.5})$$

APPENDIX F

INTERFEROGRAM OF DISTRIBUTED TARGET

Here, our analysis is restricted to the range direction. For a distributed target with a reflectivity density function denoted by $v(u)$, we assume that it can be modeled as a stationary and white process. Its autocorrelation function is then given by

$$E[v(u_1)v^*(u_2)] = \sigma^2 \delta(u_2 - u_1) \quad (\text{F.1})$$

where $\delta(\cdot)$ is the Dirac delta function, and σ^2 is the signal variance, which also represents the average reflectivity power, or Radar Cross Section (RCS) [43], per unit time. The Fourier transform of $v(u)$ is

$$V_T(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} v(u) \exp\{-j2\pi fu\} du \quad (\text{F.2})$$

where T is the size of the distributed target and should be larger than the range resolution cell size. The autocorrelation function of $V_T(f)$ is then given by

$$\begin{aligned} E[V_T(f_1)V_T^*(f_2)] &= E \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} v(u_1) \exp\{-j2\pi f_1 u_1\} du_1 \right. \\ &\quad \cdot \left. \int_{-\frac{T}{2}}^{\frac{T}{2}} v^*(u_2) \exp\{j2\pi f_2 u_2\} du_2 \right] \\ &= E \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} v(u_1)v^*(u_2) \right. \\ &\quad \cdot \exp\{-j2\pi f_1 u_1\} \exp\{j2\pi f_2 u_2\} du_1 du_2 \Big] \quad (\text{F.3}) \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} E[v(u_1)v^*(u_2)] \\ &\quad \cdot \exp\{-j2\pi f_1 u_1\} \exp\{j2\pi f_2 u_2\} du_1 du_2 \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \sigma^2 \delta(u_1 - u_2) \\ &\quad \cdot \exp\{-j2\pi f_1 u_1\} \exp\{j2\pi f_2 u_2\} du_1 du_2 \\ &= \sigma^2 \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp\{-j2\pi f_1 u_2\} \exp\{j2\pi f_2 u_2\} du_2 \\ &= \sigma^2 T \text{sinc}[(f_2 - f_1)T] \end{aligned}$$

If $T \rightarrow +\infty$

$$\begin{aligned} E[V(f_1)V^*(f_2)] &= \lim_{T \rightarrow +\infty} E[V_T(f_1)V_T^*(f_2)] \\ &= \lim_{T \rightarrow +\infty} \sigma^2 \text{sinc}[\pi(f_2 - f_1)T] \\ &= \sigma^2 \delta(f_2 - f_1) \end{aligned} \quad (\text{F.4})$$

Equation (F.4) shows that the reflectivity spectrum of the distributed target at different frequencies is uncorrelated.

For a finite target size T , the 3-dB width of the sinc function in (F.3) is $0.886/T$. Therefore, the correlation can be reasonably considered negligible when the frequency separation $|f_2 - f_1|$ exceeds $0.886/T$. As an example, consider a distributed target consisting of N range pixels. Let f_{sample} denote the range sampling rate. Then $T = N/f_{\text{sample}}$. After transforming the signal to the frequency domain via a Discrete Fourier Transform (DFT), the frequency sampling interval is $f_{\text{sample}}/N = 1/T$, which is greater than $0.886/T$. Therefore, the different frequency bins can be considered uncorrelated. In fact, the separation between different frequency bins is exactly an integer multiple of f_{sample}/N , which falls precisely at the zeros of the sinc function in (F.3), implying that different frequency bins are completely uncorrelated.

The aforementioned theoretical results can be further validated experimentally using real SAR data. To this end, we use an ALOS-1 FBD mode image acquired over the Amazon rainforest. The entire area can be approximately regarded as a single distributed target. The number of range pixels is 4640, which can also be considered as the size of the distributed target. The range sampling rate f_{sample} is 16 MHz. Following zero-padding to a length of $10N$ and a subsequent DFT in the range direction, the frequency spacing between adjacent samples is reduced to $\delta f = f_{\text{sample}}/10N = 345$ Hz, which

enables the examination of correlations between different spectral components at a finer granularity. The result is shown in Fig. F.1. The resulting autocorrelation function closely follows the sinc function given in (F.3). Furthermore, it is essentially zero at integer multiples of f_{sample}/N .

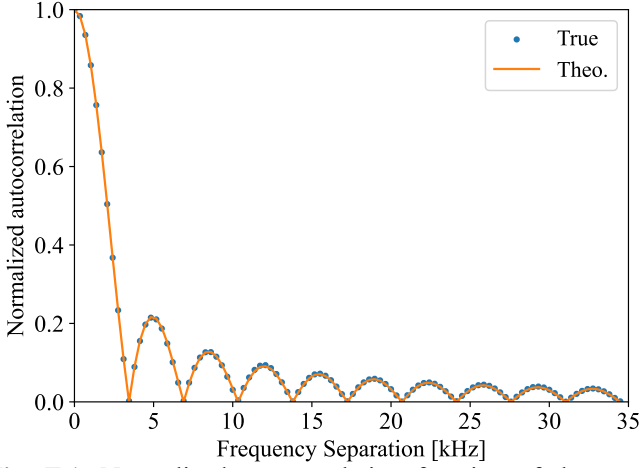


Fig. F.1. Normalized autocorrelation function of the range spectrum of an ALOS-1 FBD mode image acquired over the Amazon rainforest.

To simplify the derivation of the expectation of the interferogram, we first ignore the distinction between the slant-range and ground-range directions. Furthermore, we assume that the tropospheric and ionospheric effects remain constant within the distributed target. Under these assumptions, we can use the simplified forms of (39) and (44):

$$\begin{aligned} z_r &= \int_{D_r} V_T(f) H_r(f) df \\ z_s &= \int_{D_s} V_T(f) H_s(f) df \end{aligned} \quad (\text{F.5})$$

where D is the frequency integration interval, and $H(f) = \exp\{j\pi\mu_2 f^2\}$ is the quadratic frequency term induced by the ionosphere. Unlike $V_T(f)$, which is a stochastic process, $H(f)$ is a deterministic function. The expectation of the interferogram is then given by

$$\begin{aligned} E[z_r z_s^*] &= E \left[\int_{D_r} V_T(f) H_r(f) df \int_{D_s} V_T^*(f) H_s^*(f) df \right] \\ &= \int_{D_s} \int_{D_r} E[V_T(f_1) V_T^*(f_2)] H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \int_{D_s} \int_{D_r} \sigma^2 T \text{sinc}((f_2 - f_1)T) \cdot H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \int_{D_s} \int_{D_r} \sigma^2 \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp\{j2\pi f_1 u\} \exp\{-j2\pi f_2 u\} du \\ &\quad \cdot H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\int_{D_r} H_r(f_1) \exp\{j2\pi f_1 u\} df_1 \right] \\ &\quad \cdot \left[\int_{D_s} H_s^*(f_2) \exp\{-j2\pi f_2 u\} df_2 \right] du \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} h_r(u) h_s^*(u) du \end{aligned} \quad (\text{F.6})$$

where $h(u)$ is the inverse Fourier transform of $H(f)$ over the frequency band D . When no spectral loss occurs, $D = [-B/2, B/2]$, and $h(u)$ is given by (C.1). It is a sinc function as given in (C.3), but is slightly broadened by the quadratic frequency term. For an ideal sinc function, the main lobe contains approximately 90% of the total energy. Therefore, the result in (F.6) is approximately the same for finite and infinite T .

Thus, despite the finite extent of the distributed target, the integration limits can be treated as infinite in the derivations. As $T \rightarrow +\infty$, (F.6) reduces to a simplified form:

$$\begin{aligned} E[z_r z_s^*] &= E \left[\int_{D_r} V(f_1) H_r(f_1) df_1 \int_{D_s} V^*(f_2) H_s^*(f_2) df_2 \right] \\ &= \int_{D_s} \int_{D_r} E[V(f_1) V^*(f_2)] H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \sigma^2 \int_{D_s} \int_{D_r} \delta(f_2 - f_1) H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \sigma^2 \int_{D_s} H_r(f_2) H_s^*(f_2) \cdot \mathbf{1}_{D_r}(f_2) df_2 \\ &= \sigma^2 \int_{D_r \cap D_s} H_r(f) H_s^*(f) df \end{aligned} \quad (\text{F.7})$$

where $\mathbf{1}_{D_r}(f_2)$ is the indicator function of D_r , which equals 1 if $f_2 \in D_r$ and 0 otherwise. The result indicates that different spectral components are uncorrelated. Therefore, non-overlapping spectral components do not contribute to the expectation of the interferogram.

When wavenumber shift [1] is taken into account, (F.5) should be reformulated as

$$\begin{aligned} z_r &= \int_{D_r} V(k_r(f + f_c)) H_r(f) df \\ z_s &= \int_{D_s} V(k_s(f + f_c)) H_s(f) df \end{aligned} \quad (\text{F.8})$$

where k is the frequency conversion coefficient between the slant-range and ground-range directions, as defined in (38). Combined (41) and (45) with (F.8), the expectation of the interferogram is given by

$$\begin{aligned} E[z_r z_s^*] &= \int_{D_s} \int_{D_r} E[V(k_r(f_1 + f_c)) V^*(k_s(f_2 + f_c))] \\ &\quad \cdot H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \sigma^2 \int_{D_s} \int_{D_r} \delta(f_2 - f_1 - 2\Delta f) H_r(f_1) H_s^*(f_2) df_1 df_2 \\ &= \sigma^2 \int_{D_s} H_r(f_2 - 2\Delta f) H_s^*(f_2) \cdot \mathbf{1}_{D_r}(f_2 - 2\Delta f) df_2 \\ &= \sigma^2 \int_{(D_r + \Delta f) \cap (D_s - \Delta f)} H_r(f - \Delta f) H_s^*(f + \Delta f) df \end{aligned} \quad (\text{F.9})$$

where Δf is defined in (46). The result has essentially the same form as (F.7), implying that only the complex conjugate product of the slant-range frequency components that correspond to the same ground-range frequency contributes to the expectation of the interferogram.

APPENDIX G

GENERALIZED DERIVATION OF INTERFEROMETRIC PHASE

To calculate the interferometric phase, we retain only the overlapping spectral components in (39) and (44) and express

them as

$$\begin{aligned}
 p_r(t_r) &= \exp\{-j2\pi f_c(t_{\text{geom},r} + t_{\text{tropo},r})\} \\
 &\quad \cdot \int_{f_{\min}-\Delta f}^{f_{\max}-\Delta f} V(k_r(f_1 + f_c)) H_{\text{iono},r}(f_1 \\
 &\quad + f_c) \exp\{j2\pi f_1 \tau_r\} df_1 \\
 p_s(t_s) &= \exp\{-j2\pi f_c(t_{\text{geom},s} + t_{\text{tropo},s})\} \\
 &\quad \cdot \int_{f_{\min}+\Delta f}^{f_{\max}+\Delta f} V(k_s(f_2 + f_c)) H_{\text{iono},s}(f_2 \\
 &\quad + f_c) \exp\{j2\pi f_2 \tau_s\} df_2
 \end{aligned} \tag{G.1}$$

where t_r and t_s are the times of the reference and secondary signals, respectively, and $\tau_r = t_r - t_{\text{geom},r} - t_{\text{tropo},r}$, $\tau_s = t_s - t_{\text{geom},s} - t_{\text{tropo},s}$. Here, $f_{\min}$ denotes the average of the lower bounds of the effective frequency ranges for the reference and secondary signals, while $f_{\max}$ denotes the average of their upper bounds. Taking wavenumber shift into account, $f_{\min} = -B/2 + |\Delta f|$ and $f_{\max} = B/2 - |\Delta f|$.

Using the result in (F.9) and introducing a frequency variable f defined by $f = f_2 - \Delta f = f_1 + \Delta f$, we obtain

$$\begin{aligned}
 E[P_r(t_r) \cdot P_s^*(t_s)] &= \sigma^2 \exp\{-j2\pi f_c(\Delta t_{\text{geom}} + \Delta t_{\text{tropo}})\} \\
 &\quad \cdot \int_{f_{\min}}^{f_{\max}} H_{\text{iono},r}[f + (f_c - \Delta f)] H_{\text{iono},s}^*[f \\
 &\quad + (f_c + \Delta f)] \exp\{j2\pi[(f - \Delta f)\tau_r \\
 &\quad - (f + \Delta f)\tau_s]\} df
 \end{aligned} \tag{G.2}$$

We first examine the component associated with the reference signal in the integrand of (G.2):

$$\begin{aligned}
 H_{\text{iono},r}[f + (f_c - \Delta f)] \exp\{j2\pi(f - \Delta f)\tau_r\} \\
 = \exp\left\{j\left[\frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)} - \frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)^2} f \right. \right. \\
 \left. \left. + \frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)^3} f^2 + 2\pi(f - \Delta f)\tau_r\right]\right\}
 \end{aligned} \tag{G.3}$$

The phase term in the exponent can be decomposed into constant, linear, and quadratic components with respect to frequency, denoted by A_0 , A_1 , and A_2 , respectively, and given by

$$\begin{aligned}
 A_0(\text{TEC}_r, -\Delta f, \tau_r) &= \frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)} - 2\pi\Delta f\tau_r \\
 A_1(\text{TEC}_r, -\Delta f, \tau_r) &= -\frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)^2} + 2\pi\tau_r \\
 A_2(\text{TEC}_r, -\Delta f, \tau_r) &= \frac{4\pi K_2 \cdot \text{TEC}_r}{c(f_c - \Delta f)^3}
 \end{aligned} \tag{G.4}$$

A similar decomposition can be performed for the component associated with the secondary signal.

Defining $C_i = A_i(\text{TEC}_r, -\Delta f, \tau_r) - A_i(\text{TEC}_s, \Delta f, \tau_s)$ for $i = 0, 1, 2$, the integrand in (G.2) can be expressed as $\exp\{jC_0 + jC_1f + jC_2f^2\}$. Given that C_1f and C_2f^2 are small quantities, the integral can be approximated as follows:

$$\begin{aligned}
 &\int_{f_{\min}}^{f_{\max}} \exp\{jC_0 + jC_1f + jC_2f^2\} df \\
 &= \exp\{jC_0\} \int_{f_{\min}}^{f_{\max}} \exp\{jC_1f + jC_2f^2\} df \\
 &\approx \exp\{jC_0\} \int_{f_{\min}}^{f_{\max}} [1 + j(C_1f + C_2f^2)] df \\
 &= \exp\{jC_0\} \cdot \left\{ f_{\max} - f_{\min} \right. \\
 &\quad \left. + j\left[\frac{C_1}{2}(f_{\max}^2 - f_{\min}^2) + \frac{C_2}{3}(f_{\max}^3 - f_{\min}^3)\right] \right\}
 \end{aligned} \tag{G.5}$$

Consequently, the interferometric ionospheric phase is given by

$$\begin{aligned}
 \Delta\varphi_{\text{iono}} &= C_0 + \arctan \frac{\frac{C_1}{2}(f_{\max}^2 - f_{\min}^2) + \frac{C_2}{3}(f_{\max}^3 - f_{\min}^3)}{f_{\max} - f_{\min}} \\
 &\approx C_0 + \frac{C_1}{2}(f_{\min} + f_{\max}) + \frac{C_2}{3}(f_{\min}^2 + f_{\min}f_{\max} + f_{\max}^2)
 \end{aligned} \tag{G.6}$$

It follows from (G.6) that, when the integration interval is symmetric, i.e., $f_{\min} + f_{\max} = 0$, the contribution of the linear term C_1f in the integrand of (G.5) to the interferometric ionospheric phase is approximately zero.

If the reference and secondary signals are coregistered at the target positions of the full spectra, i.e., $\tau_r = 2K_2\text{TEC}_r/cf_c^2$ and $\tau_s = 2K_2\text{TEC}_s/cf_c^2$, the result in (48) is obtained. If the reference and secondary signals are coregistered at the target positions of the overlapping spectra, i.e., $\tau_r = 2K_2\text{TEC}_r/c(f_c - \Delta f)^2$ and $\tau_s = 2K_2\text{TEC}_s/c(f_c + \Delta f)^2$, the result in (49) is obtained.

For the first scenario in Section IV-A, the lower limit of the integration interval, $f_{\min}$, should be replaced with $f_{\text{sub}} - f_c - B_{\text{sub}}/2 + |\Delta f|$, and the upper limit, $f_{\max}$, should be replaced with $f_{\text{sub}} - f_c + B_{\text{sub}}/2 - |\Delta f|$. For the second scenario, the center frequency f_c should be replaced with f_{sub} ; the lower limit, $f_{\min}$, should be replaced with $-B_{\text{sub}}/2 + |\Delta f|$; and the upper limit, $f_{\max}$, should be replaced with $B_{\text{sub}}/2 - |\Delta f|$. These substitutions yield the results in (53) and (54).

When coregistration misalignments are incorporated, i.e., $\tau_r = 2K_2\text{TEC}_r/cf_c^2 + \Delta\tau_r$ and $\tau_s = 2K_2\text{TEC}_s/cf_c^2 + \Delta\tau_s$, only the coefficients C_0 and C_1 are modified, whereas C_2 remains unchanged. Then the interferometric ionospheric phase becomes

$$\begin{aligned}
 \Delta\varphi'_{\text{iono}} &= \Delta\varphi_{\text{iono}} - 2\pi\Delta f(\Delta\tau_r + \Delta\tau_s) \\
 &\quad + \pi(f_{\min} + f_{\max})(\Delta\tau_r - \Delta\tau_s)
 \end{aligned} \tag{G.7}$$

Coregistration misalignment gives rise to two phase terms. The first term is associated with the interferometric effective center frequency and can be further decomposed into the contributions of absolute and relative misalignments, as discussed in Section III-D. The second term is associated with the actual center frequency; it is independent of absolute misalignment and depends solely on relative misalignment. In general, $f_{\min} + f_{\max} = 0$; this term needs to be accounted for only when sub-band filtering is performed without shifting the center frequency, as described in Section IV-E.

APPENDIX H

IMPACTS OF THE PVP ON INSAR TIME SERIES

Here, we derive the impact of the PVP on the InSAR time series estimated by least squares when using the range split-spectrum method to estimate the InSAR ionospheric phase. Assume that there are n SAR images, with the first image serving as the reference image, and all remaining images coregistered to it. In fact, the choice of the reference image does not affect the subsequent derivation. Furthermore, assume that the network is fully connected. We focus on a

specific pixel within the image stack. Let $B_{\perp,i}$ ($1 \leq i \leq n$) denote the perpendicular baseline of the i -th image relative to the reference image. Following (46) and (60), the PVP contribution in the ionospheric phase estimated from images i and j is given by

$$\begin{aligned} \phi_{ij} &= \frac{4\pi K_2(\text{TEC}_i + \text{TEC}_j)}{cf_c^2} \Delta f_{ij} \\ &\approx \frac{2\pi K_2(\text{TEC}_i + \text{TEC}_j)(B_{\perp,i} - B_{\perp,j})}{cf_c^2 R \tan(\theta - \alpha)} \end{aligned} \quad (\text{H.1})$$

where Δf_{ij} represents half of the spectral shift, θ is the look angle of the pixel in the reference image, R is the corresponding slant range, and α is the terrain slope.

For a fully connected network, a system of linear equations can be established as follows:

$$x_i - x_j = \frac{\phi_{ij}}{2\pi} \cdot \frac{\lambda}{2} (1 \leq i < j \leq n) \quad (\text{H.2})$$

where x_k ($1 \leq k \leq n$) is the displacement error of the k -th image, and λ is the wavelength. The system can be written in matrix form as $AX = Y$, where A is the design matrix. To estimate X , we impose the constraint $\sum_{k=1}^n x_k = 0$. For simplicity, we consider an unweighted least-squares estimation, leading to

$$\begin{aligned} x_k &= \frac{\lambda}{4\pi n} \left(\sum_{j>k} \phi_{kj} - \sum_{i<k} \phi_{ik} \right) \\ &= \frac{\gamma}{n} (n\text{TEC}_k B_{\perp,k} + B_{\perp,k} \sum \text{TEC}_l - \text{TEC}_k \sum B_{\perp,l} - \sum \text{TEC}_l B_{\perp,l}) \\ &= \gamma \left(\text{TEC}_k B_{\perp,k} + \overline{\text{TEC}} \cdot B_{\perp,k} - \overline{B_{\perp}} \cdot \text{TEC}_k - \frac{\sum \text{TEC}_l B_{\perp,l}}{n} \right) \end{aligned} \quad (\text{H.3})$$

where the overbar denotes the temporal average, and

$$\gamma = \frac{K_2 \lambda}{2cf_c^2 R \tan(\theta - \alpha)} \quad (\text{H.4})$$

We first examine the impact of temporal variations in the perpendicular baseline by assuming a constant TEC over time. Then $x_k = 2\gamma \overline{\text{TEC}} (B_{\perp,k} - \overline{B_{\perp}})$. If the perpendicular baselines exhibit temporal correlation, the PVP allocated to individual SAR images through least-squares adjustment will likewise exhibit temporal correlation, thereby introducing a bias of $2\gamma \overline{\text{TEC}} \cdot v_{B_{\perp}}$ into the estimated displacement velocity, where $v_{B_{\perp}}$ is the temporal variation rate of the perpendicular baseline.

In fact, the TEC exhibits temporal variations associated with the solar cycle. The temporal trends of the TEC and the perpendicular baseline are coupled in (H.3). Assuming that the n images are acquired consecutively with a revisit period of M days, a linear fit to the displacement errors $\{x_k\}$ yields the estimated velocity error:

$$\bar{v} = \frac{\gamma}{M \sum c_k^2} (\sum c_k \text{TEC}_k B_{\perp,k} + \overline{\text{TEC}} \sum c_k B_{\perp,k} - \overline{B_{\perp}} \sum c_k \text{TEC}_k) \quad (\text{H.5})$$

where $c_k = k - (n+1)/2$.

For further simplification, we assume that both the TEC and the perpendicular baseline vary linearly with time, with respective variation rates v_{TEC} and $v_{B_{\perp}}$, i.e., $\text{TEC}_k = \overline{\text{TEC}} + M v_{\text{TEC}} c_k$, $B_{\perp,k} = \overline{B_{\perp}} + M v_{B_{\perp}} c_k$. The resulting velocity error is $\bar{v} = 2\gamma \overline{\text{TEC}} \cdot v_{B_{\perp}}$, which depends solely on the temporal trend of the perpendicular baseline and is independent of the

temporal trend of the TEC.

In practice, the temporal variations of the TEC and the perpendicular baseline may be more complex. Moreover, the least-squares estimation may be performed over a partially connected network with a weighting matrix. Nevertheless, the results in this appendix provide useful insights into the impacts of the PVP on the InSAR time series.

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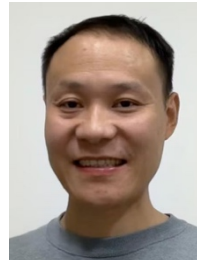
REFERENCES

- [1] F. Gatelli, A. M. Guarnieri, F. Parizzi, P. Pasquali, C. Prati, and F. Rocca, "The wavenumber shift in SAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 32, no. 4, pp. 855-865, Jul. 1994, doi: 10.1109/36.298013.
- [2] M. A. Martin Serrano, I. Shurmer, and X. Marc, "Sentinel-1: Operational approach to the orbit control strategy," in *Proc. 23rd Int. Symp. Space Flight Dyn.*, 2012.
- [3] K. Akiyama, H. Itoh, H. Masuda, S. Kasho, and T. Sakamoto, "Operational validation of precise orbit determination for ALOS-2," in *Proc. 14th Int. Conf. Space Oper.*, 2016, pp. 2521-2521.
- [4] B. Chapman, P. Rosen, S. Hensley, and S. Buckley, "Calibration and validation plan for the NASA-ISRO synthetic aperture radar (NISAR)," in *Proc. URSI Asia-Pacific Radio Sci. Conf.*, New Delhi, India, 2019, pp. 1-1, doi: 10.23919/URSIAP-RASC.2019.8738266.
- [5] R. Wang *et al.*, "LuTan-1: An innovative L-band spaceborne bistatic interferometric synthetic aperture radar mission," *IEEE Geosci. Remote Sens. Mag.*, vol. 13, no. 2, pp. 58-78, Jun. 2025, doi: 10.1109/MGRS.2024.3478761.
- [6] M. Spencer and F. Ulaby, "Spectrum issues faced by active remote sensing: Radio frequency interference and operational restrictions," *IEEE Geosci. Remote Sens. Mag.*, vol. 4, no. 1, pp. 40-45, Mar. 2016, doi: 10.1109/MGRS.2016.2517410.
- [7] H. Xu, Z. Wu, W. Liu, J. Li, and Q. Feng, "Analysis of the Effect of Interference on InSAR," *IEEE Sensors J.*, vol. 15, no. 10, pp. 5659-5668, Oct. 2015, doi: 10.1109/JSEN.2015.2445931.
- [8] M. Tao, J. Su, Y. Huang, and L. Wang, "Mitigation of radio frequency interference in synthetic aperture radar data: Current status and future trends," *Remote Sens.*, vol. 11, no. 20, Art. no. 2438, Oct. 2019, doi: 10.3390/rs11202438.
- [9] F. J. Meyer, J. B. Nicoll, and A. P. Doulgeris, "Correction and characterization of radio frequency interference signatures in L-band synthetic aperture radar data," *IEEE Trans. Geosci. Remote Sens.*, vol. 51, no. 10, pp. 4961-4972, Oct. 2013, doi: 10.1109/TGRS.2013.2252469.
- [10] R. Natsuaki, T. Motohka, M. Watanabe, M. Shimada, and S. Suzuki, "An autocorrelation-based radio frequency interference detection and removal method in azimuth-frequency domain for SAR image," *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.*, vol. 10, no. 12, pp. 5736-5751, Dec. 2017, doi: 10.1109/JSTARS.2017.2775205.
- [11] *Sentinel-1 Level 1 Detailed Algorithm Definition*, Issue 2, Rev. 5, European Space Agency, Noordwijk, The Netherlands, Nov. 2022. [Online]. Available: <https://sentiwiki.copernicus.eu/web/document-library>.
- [12] J. Chen *et al.*, "Characterization and mitigation of radio frequency interference signatures in L-band LuTan-1 InSAR system: First results and assessment," *IEEE Trans. Radar Syst.*, vol. 2, pp. 404-420, 2024, doi: 10.1109/TRS.2024.3385181.
- [13] D. Petrolati *et al.*, "An Overview of the Copernicus Rose-L SAR Instrument," in *IGARSS 2023 - 2023 IEEE International Geoscience and*

- Remote Sensing Symposium*, Pasadena, CA, USA, 2023, pp. 4310-4313, doi: 10.1109/IGARSS52108.2023.10281670.
- [14] S. N. Madsen, "Absolute phase determination techniques in SAR interferometry," in *Proc. SPIE Symp. OE/Aerosp. Sens. Dual-Use Photon.*, Orlando, FL, USA, 1995, vol. 2487, pp. 393-401.
- [15] R. Scheiber and A. Moreira, "Coregistration of interferometric SAR images using spectral diversity," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 5, pp. 2179-2191, Sep. 2000, doi: 10.1109/36.868876.
- [16] G. Gomba, A. Parizzi, F. De Zan, M. Eineder, and R. Bamler, "Toward operational compensation of ionospheric effects in SAR interferograms: The split-spectrum method," *IEEE Trans. Geosci. Remote Sens.*, vol. 54, no. 3, pp. 1446-1461, Mar. 2016, doi: 10.1109/TGRS.2015.2481079.
- [17] K. P. Papathanassiou and S. R. Cloude, "Single-baseline polarimetric SAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 39, no. 11, pp. 2352-2363, Nov. 2001, doi: 10.1109/36.964971.
- [18] R. F. Hanssen, *Radar Interferometry: Data Interpretation and Error Analysis*, 1st ed. Dordrecht, The Netherlands: Springer, 2001.
- [19] N. Setiawan and M. Furuuya, "Tropospheric dispersive phase anomalies during heavy rain detected by L-band InSAR and their interpretation," *Earth Planets Space*, vol. 73, no. 1, Art. no. 145, Dec. 2021, doi: 10.1186/s40623-021-01470-9.
- [20] C. Liang, E. J. Fielding, Z. Liu, T. Motohka, R. Natsuaki, and S.-H. Yun, "An analysis of the potentials of L-band SAR satellites for measuring azimuth motion," *Remote Sens. Environ.*, vol. 315, Art. no. 114426, Dec. 2024, doi: 10.1016/j.rse.2024.114426.
- [21] B. Lundborg and M. Lundgren, "On the spectral width of chirpsounder signals," *J. Atmos. Terr. Phys.*, vol. 54, no. 3-4, pp. 311-321, Mar.-Apr. 1992, doi: 10.1016/0021-9169(92)90010-1.
- [22] S. Salous, "Dispersion of chirp pulses by the ionosphere," *J. Atmos. Terr. Phys.*, vol. 56, no. 8, pp. 979-994, Jun. 1994, doi: 10.1016/0021-9169(94)90158-9.
- [23] D. Perissin, C. Prati, M. E. Engdahl, and Y. L. Desnos, "Validating the SAR wavenumber shift principle with the ERS-Envisat PS coherent combination," *IEEE Trans. Geosci. Remote Sens.*, vol. 44, no. 9, pp. 2343-2351, Sep. 2006, doi: 10.1109/TGRS.2006.875455.
- [24] G. Fornaro and A. M. Guarnieri, "Minimum mean square error space-varying filtering of interferometric SAR data," *IEEE Trans. Geosci. Remote Sens.*, vol. 40, no. 1, pp. 11-21, Jan. 2002, doi: 10.1109/36.981345.
- [25] F. Yang, C. Liang, and Y. Wang, "InSAR phase errors due to range misregistrations," *IEEE Trans. Geosci. Remote Sens.*, early access, doi: 10.1109/TGRS.2026.3664181.
- [26] R. Bamler and M. Eineder, "Accuracy of differential shift estimation by correlation and split-bandwidth interferometry for wideband and delta-k SAR systems," *IEEE Geosci. Remote Sens. Lett.*, vol. 2, no. 2, pp. 151-155, Apr. 2005, doi: 10.1109/LGRS.2004.843203.
- [27] T. Motohka, Y. Kankaku, S. Miura, and S. Suzuki, "ALOS-4 L-band SAR mission and observation," in *Proc. IEEE Int. Geosci. Remote Sens. Symp.*, Yokohama, Japan, 2019, pp. 5271-5273, doi: 10.1109/IGARSS.2019.8898169.
- [28] P. A. Rosen *et al.*, "The NASA-ISRO SAR mission: A summary," *IEEE Geosci. Remote Sens. Mag.*, vol. 13, no. 2, pp. 8-34, Jun. 2025, doi: 10.1109/MGRS.2025.3578258.
- [29] U. Wegmüller, C. Werner, O. Frey, C. Magnard, and T. Strozzi, "Reformulating the split-spectrum method to facilitate the estimation and compensation of the ionospheric phase in SAR interferograms," *Procedia Comput. Sci.*, vol. 138, pp. 318-325, 2018, doi: 10.1016/j.procs.2018.10.045.
- [30] S. Samsonov, "Topographic correction for ALOS PALSAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 7, pp. 3020-3027, Jul. 2010, doi: 10.1109/TGRS.2010.2043739.
- [31] H. Fattahi and F. Amelung, "DEM error correction in InSAR time series," *IEEE Trans. Geosci. Remote Sens.*, vol. 51, no. 7, pp. 4249-4259, Jul. 2013, doi: 10.1109/TGRS.2012.2227761.
- [32] R. Brcic, M. Eineder, and R. Bamler, "Interferometric absolute phase determination with TerraSAR-X wideband SAR data," in *Proc. IEEE Radar Conf.*, Pasadena, CA, USA, 2009, pp. 1-6, doi: 10.1109/RADAR.2009.4977112.
- [33] H. Hirano, O. Isoguchi, T. Motohka, M. Ohki, and T. Tadono, "Ionospheric effects and correction studies on ALOS-2 PALSAR-2 data," presented at the CEOS WGC SAR Workshop, Oberpfaffenhofen, Germany, Oct. 2023. [Online]. Available: https://calvalportal.ceos.org/html/portal/sarc/documents/documents/26_Hirano_Hirano_1570933421_presentation.pdf
- [34] C. Liang and E. J. Fielding, "Interferometry with ALOS-2 full-aperture ScanSAR data," *IEEE Trans. Geosci. Remote Sens.*, vol. 55, no. 5, pp. 2739-2750, May 2017, doi: 10.1109/TGRS.2017.2653190.
- [35] C. Liang and E. J. Fielding, "Measuring Azimuth Deformation With L-Band ALOS-2 ScanSAR Interferometry," in *IEEE Transactions on Geoscience and Remote Sensing*, vol. 55, no. 5, pp. 2725-2738, May 2017, doi: 10.1109/TGRS.2017.2653186.
- [36] P. A. Rosen, E. Gurrola, G. F. Sacco, and H. Zebker, "The InSAR scientific computing environment," in *Proc. Eur. Conf. Synth. Aperture Radar*, Nuremberg, Germany, 2012, pp. 730-733.
- [37] L. Martire *et al.*, "The JPL-GIM algorithm and products: Multi-GNSS high-rate global mapping of total electron content," *J. Geod.*, vol. 98, no. 44, 2024, doi: 10.1007/s00190-024-01860-3.
- [38] X. Li, C. Liang, J. Li, and K. Chen, "An evaluation of Global Ionosphere Maps for the ionospheric correction of C-band InSAR data," *J. Geod.*, accepted for publication.
- [39] I. G. Cumming and F. H. Wong, *Digital Processing of Synthetic Aperture Radar Data*, 1st ed. Norwood, MA, USA: Artech House, 2005.
- [40] E. Petrie and M. Hernandez, "Models for atmospheric propagation delays," in *IERS Conventions (2010)*, Frankfurt am Main, Germany: IERS Central Bureau, 2012, pp. 132-150.
- [41] C. Wang, L. Chen, and L. Liu, "A new analytical model to study the ionospheric effects on VHF/UHF wideband SAR imaging," *IEEE Trans. Geosci. Remote Sens.*, vol. 55, no. 8, pp. 4545-4557, Aug. 2017, doi: 10.1109/TGRS.2017.2693396.
- [42] A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals & Systems*, 2nd ed. Upper Saddle River, NJ, USA: Prentice Hall, 1997.
- [43] C. Oliver and S. Quegan, *Understanding Synthetic Aperture Radar Images*. Raleigh, NC, USA: SciTech Publishing, 2004, pp. 29-30.



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