

InSAR and Closure Phase Errors Due to the Along-Track Ionospheric Variations Within the Synthetic Aperture

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Abstract—With the launch of a fleet of L-band missions, we have entered the era of L-band spaceborne synthetic aperture radar (SAR). The ionosphere is a significant issue for long-wavelength spaceborne SAR systems. In this work, we systematically investigate the impacts of along-track ionospheric variations within the synthetic aperture. We expand the ionospheric total electron content (TEC) using a Taylor series about the Doppler centroid time and analyze the effects of each component on the focusing of a point target. We find that the odd-order ionospheric phase components mainly induce a shift in the peak position, whereas the even-order ionospheric phase components mainly induce a phase error; in addition to the well-recognized defocusing effect. Together with the linear phase induced by the Doppler centroid frequency, these effects can lead to significant SAR interferometry (InSAR) and closure phase errors depending on the ionospheric activity level. It is demonstrated that precise azimuth coregistration can effectively mitigate the closure phase errors.

Index Terms—Azimuth misregistration, closure phase, InSAR, ionosphere.

I. INTRODUCTION

THE last decade has seen a rapid expansion of L-band spaceborne synthetic aperture radar (SAR) missions, including ALOS-2 [1], SAOCOM-1A/B, LuTan-1A/B [2], ALOS-4 [3], and NISAR [4]. This fleet of L-band missions is poised to grow further, with upcoming missions such as ROSE-L under active development. For SAR interferometry (InSAR), L-band substantially improves coherence, a persistent bottleneck for X- and C-band systems, thereby benefiting a variety of applications. The enhanced coherence is particularly vital for the analysis of closure phase, a recently identified observable [5] that has the potential to retrieve physical parameters, such as soil moisture [6], [7], [8], and that plays a key role in mitigating systematic errors in InSAR time series [9].

A most significant challenge for L-band spaceborne SAR missions is their high susceptibility to ionospheric disturbances. A brief summary of the resulting ionospheric effects

on SAR imaging is provided in [10]. In the azimuth direction, radar signals within the synthetic aperture traverse spatially varying regions of the ionosphere, leading to complex ionospheric effects. The ionosphere within the synthetic aperture can be decomposed into various phase components, among which the most prominent constant component [11] can be well estimated using the range split-spectrum method [11], [12], [13].

Regarding the higher order ionospheric phase components within the synthetic aperture, pixel shifts induced by the linear ionospheric phase component have long been observed in cross correlation offsets and multiple aperture InSAR (MAI) interferograms [14], [15], [16], [17], [18], [19]. They have been mitigated to improve the retrieval of azimuth motions [10], [16], [20], [21], [22], albeit with large uncertainties. The quadratic and higher order ionospheric phase components, however, remain poorly characterized in the existing literature. To date, their effects have predominantly been described in qualitative terms such as defocusing or spatial resolution degradation [14], [23], [24], [25]. This lack of quantification has hindered the assessment of the resulting InSAR phase errors.

In this work, we aim to comprehensively and systematically characterize the effects of along-track ionospheric variations within the synthetic aperture. We begin with a Taylor series expansion of the ionospheric total electron content (TEC) about the Doppler centroid time and then formulate the effects of each ionospheric phase component on point target imaging in Section II. Building on these effects, we derive the resulting InSAR phase errors for both point and distributed targets in Section III. Notably, we find that these effects also induce closure phase errors. Subsequently, we validate our findings through numerical simulations in Section IV and real-data experiments in Section V. Finally, Section VI summarizes our conclusions.

II. EFFECTS ON POINT TARGET IMAGING

A. Background

Since the associated range–azimuth coupling effects are generally negligible, as demonstrated in Appendix A, we will focus our analysis exclusively on the azimuth direction. Under the assumption of a small squint angle and a limited synthetic aperture, the azimuth signal from a point target can

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be approximated as a linear frequency-modulated (FM) signal [26]

$$s_{az}(\eta) = A_0 w_a(\eta - \eta_c) \exp \left\{ -j\pi \frac{2V_r^2}{\lambda R_0} \eta^2 \right\} \quad (1)$$

where A_0 is the complex reflectivity of the target, η is the azimuth time referenced to zero Doppler time η_0 , η_c is the beam center crossing time or the Doppler centroid time, V_r is the effective radar velocity, λ is the radar wavelength, and R_0 is the range of closest approach. $w_a(\cdot)$ denotes the azimuth envelope due to the two-way antenna pattern and is given by

$$w_a(\eta - \eta_c) \approx \text{sinc}^2 \left\{ \frac{0.886 V_g (\eta - \eta_c)}{\theta_B R_0} \right\} \quad (2)$$

where V_g is the radar beam velocity on the ground, and θ_B is the 3-dB beamwidth of the antenna in azimuth. The azimuth FM rate, denoted by K_a , is

$$K_a = \frac{2V_r^2}{\lambda R_0}. \quad (3)$$

Virtually all existing satellite SAR systems satisfy the assumptions of a small squint angle and a limited synthetic aperture adopted in this study, at least within the context of our analysis. By expressing the azimuth signal as in (1), we have defined the origin of the azimuth time as the zero Doppler time, η_0 , which is assumed throughout this article. For the numerical analysis in the remainder of this article, we will use the parameters of an L-band satellite SAR system, where K_a is 600 Hz/s, the azimuth bandwidth is 2000 Hz, and the pulse repetition frequency (PRF) is 2400 Hz. The Doppler centroid frequency varies within ± 100 Hz.

The azimuth signal can be focused by a matched filter derived in either the time domain or the frequency domain. As demonstrated in Appendix B, the two matched filters lead to the same output. The peak position of the focused target is located at its zero Doppler time η_0 . For simplicity, the matched filter derived in the frequency domain will be employed in the subsequent analysis.

When the radar signal traverses the ionosphere, its phase is advanced by [23]

$$\phi_{\text{ion}} = \frac{4\pi K \text{TEC}(\eta)}{c f_0} \quad (4)$$

where $\text{TEC}(\eta)$ represents the TEC integrated along the signal path, $K = 40.31 \text{ m}^3/\text{s}^2$, c is the speed of light in vacuum, and f_0 is the radar carrier frequency. Then the azimuth SAR signal becomes

$$s_{az}(\eta) = A_0 w_a(\eta - \eta_c) \exp \left\{ -j\pi \frac{2V_r^2}{\lambda R_0} \eta^2 \right\} \exp \{ j\phi_{\text{ion}} \}. \quad (5)$$

Within the synthetic aperture, $\text{TEC}(\eta)$ can be expanded in a Taylor series about η_c

$$\begin{aligned} \text{TEC}(\eta) &= \text{TEC}(\eta_c) + \left. \frac{\partial \text{TEC}}{\partial \eta} \right|_{\eta=\eta_c} (\eta - \eta_c) \\ &+ \frac{1}{2} \left. \frac{\partial^2 \text{TEC}}{\partial \eta^2} \right|_{\eta=\eta_c} (\eta - \eta_c)^2 \\ &+ \frac{1}{6} \left. \frac{\partial^3 \text{TEC}}{\partial \eta^3} \right|_{\eta=\eta_c} (\eta - \eta_c)^3 \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{24} \left. \frac{\partial^4 \text{TEC}}{\partial \eta^4} \right|_{\eta=\eta_c} (\eta - \eta_c)^4 + \dots \\ &+ \frac{1}{n!} \left. \frac{\partial^n \text{TEC}}{\partial \eta^n} \right|_{\eta=\eta_c} (\eta - \eta_c)^n + \dots \end{aligned} \quad (6)$$

Substituting (6) into (4) expands the ionospheric phase term into components of different orders. We first denote the coefficients of the constant, linear, quadratic, cubic, and quartic ionospheric phase components as ϕ_{ion} , f_{ion} , K_{ion} , δ_{ion} , and ε_{ion} , respectively, such that

$$\begin{cases} \phi_{\text{ion}} = \frac{4\pi K}{c f_0} \text{TEC}(\eta_c) \\ f_{\text{ion}} = \left. \frac{2K}{c f_0} \frac{\partial \text{TEC}}{\partial \eta} \right|_{\eta=\eta_c} \\ K_{\text{ion}} = \left. \frac{2K}{c f_0} \frac{\partial^2 \text{TEC}}{\partial \eta^2} \right|_{\eta=\eta_c} \\ \delta_{\text{ion}} = \left. \frac{2K}{3c f_0} \frac{\partial^3 \text{TEC}}{\partial \eta^3} \right|_{\eta=\eta_c} \\ \varepsilon_{\text{ion}} = \left. \frac{K}{6c f_0} \frac{\partial^4 \text{TEC}}{\partial \eta^4} \right|_{\eta=\eta_c} \end{cases} \quad (7)$$

Accordingly, we define $\phi_{\text{ion}}^{(n)}$ as the n th order ionospheric phase. By combining (4), (6), and (7), the ionospheric phase components up to the fourth order are

$$\begin{cases} \phi_{\text{ion}}^{(0)} = \phi_{\text{ion}} \\ \phi_{\text{ion}}^{(1)} = \left. \frac{4\pi K}{c f_0} \frac{\partial \text{TEC}}{\partial \eta} \right|_{\eta=\eta_c} (\eta - \eta_c) = 2\pi f_{\text{ion}} (\eta - \eta_c) \\ \phi_{\text{ion}}^{(2)} = \left. \frac{4\pi K}{c f_0} \frac{1}{2} \frac{\partial^2 \text{TEC}}{\partial \eta^2} \right|_{\eta=\eta_c} (\eta - \eta_c)^2 = \pi K_{\text{ion}} (\eta - \eta_c)^2 \\ \phi_{\text{ion}}^{(3)} = \left. \frac{4\pi K}{c f_0} \frac{1}{6} \frac{\partial^3 \text{TEC}}{\partial \eta^3} \right|_{\eta=\eta_c} (\eta - \eta_c)^3 = \pi \delta_{\text{ion}} (\eta - \eta_c)^3 \\ \phi_{\text{ion}}^{(4)} = \left. \frac{4\pi K}{c f_0} \frac{1}{24} \frac{\partial^4 \text{TEC}}{\partial \eta^4} \right|_{\eta=\eta_c} (\eta - \eta_c)^4 = \pi \varepsilon_{\text{ion}} (\eta - \eta_c)^4 \end{cases} \quad (8)$$

In the context of InSAR, the difference in the constant ionospheric phase component in the synthetic aperture between two acquisitions can be well estimated by the range split-spectrum method developed in recent years [11], [12], [13]. The remaining ionospheric phase components, however, often give rise to artifacts in the imaging results that have not been well resolved or understood. In Sections II-B–II-D, we will systematically analyze the effects of each ionospheric phase component on the imaging result.

B. Effects of Linear Ionospheric Phase Component

The effects of the linear ionospheric phase component were well observed in previous works [14], [15], [16], [17], [18], [19]. Intuitively, a linear phase shifts the vertex of the original parabolic phase, as shown in Fig. 1, which is the zero Doppler time of the point target and consequently shifts the target position after focusing. The second effect is a variation in the Doppler centroid frequency. However, this variation is very small, typically below 1 Hz. The third effect is an additional phase at the vertex. To comprehensively express the effects in the focused result within a unified framework and facilitate subsequent analysis, we will rederive them in this work.

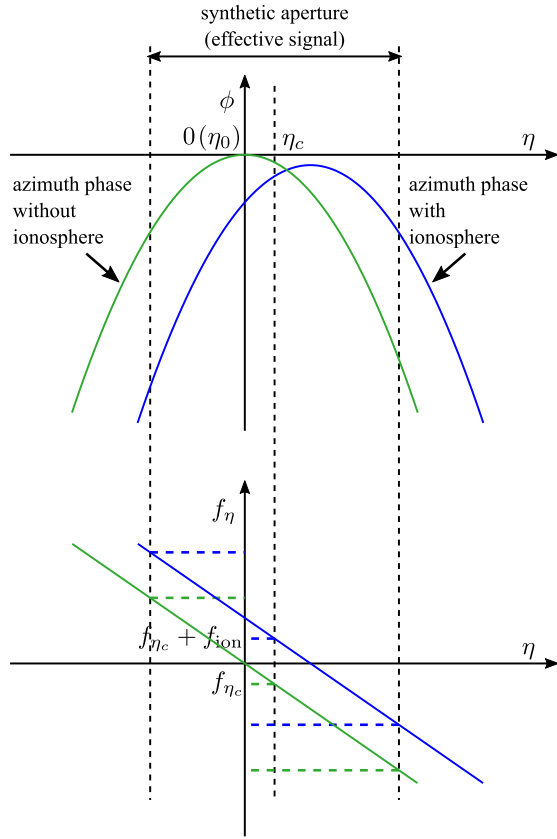


Fig. 1. Effects of the linear ionospheric phase component on the azimuth SAR signal before focusing.

Considering the linear ionospheric phase component, the azimuth SAR signal can be expressed as

$$s_{az}^{(1)}(\eta) = A_0 w_a(\eta - \eta_c) \exp\{-j\pi K_a \eta^2\} \times \exp\{j2\pi f_{ion}(\eta - \eta_c)\}. \quad (9)$$

By applying the principle of stationary phase (POSP) [27], its spectrum can be derived as

$$S_{az}^{(1)}(f_\eta) = A_0 W_a(f_\eta - f_{\eta_c} - f_{ion}) \times \exp\left\{j\pi \frac{(f_\eta - f_{ion})^2}{K_a}\right\} \exp\left\{j2\pi \frac{f_{\eta_c} f_{ion}}{K_a}\right\} \quad (10)$$

where $f_{\eta_c} = -K_a \eta_c$ is the original Doppler centroid frequency without considering the ionospheric phase, and $W_a(f_\eta - f_{\eta_c} - f_{ion})$ represents the envelope of the azimuth spectrum, which is centered at $f_{\eta_c} + f_{ion}$. Here, the constant term resulting from POSP is omitted.

The matched filter, which ignores the ionospheric phase, is constructed by taking the complex conjugate of the quadratic phase term in $S_{az}^{(1)}(f_\eta)$ (see Appendix B for details)

$$H_{az}(f_\eta) = \text{rect}\left[\frac{(f_\eta - f_{\eta_c})}{B}\right] \exp\left\{-j\pi \frac{f_\eta^2}{K_a}\right\} \quad (11)$$

where $\text{rect}(\cdot)$ is the rectangular function, and B is the azimuth bandwidth.

The spectrum after applying this matched filter is

$$S_{az,\text{focused}}^{(1)}(f_\eta)$$

$$= S_{az}(f_\eta) \cdot H_{az}(f_\eta) \approx A_0 W_a\left(f_\eta - f_{\eta_c} - \frac{f_{ion}}{2}\right) \times \exp\left\{j\pi \left[\frac{-2f_\eta f_{ion} + f_{ion}^2 + 2f_{\eta_c} f_{ion}}{K_a}\right]\right\}. \quad (12)$$

The time-domain focusing result is obtained by an inverse Fourier transform, yielding

$$s_{az,\text{focused}}^{(1)}(\eta) \approx A_0 \text{sinc}\left\{B\left(\eta - \frac{f_{ion}}{K_a}\right)\right\} \times \exp\left\{j2\pi\left(f_{\eta_c} + \frac{f_{ion}}{2}\right)\eta\right\}. \quad (13)$$

In the presence of the linear ionospheric phase component, three effects can be observed as follows.

- 1) The Doppler centroid frequency is shifted by $f_{ion}/2$, but this shift is very small, typically below 1 Hz.
- 2) The peak position is shifted by

$$\eta_f = \frac{f_{ion}}{K_a}. \quad (14)$$

- 3) A linear phase, corresponding to the shifted Doppler centroid frequency, is induced. The resulting phase is

$$2\pi \frac{f_{ion}}{2} \eta = \pi f_{ion} \eta. \quad (15)$$

Note that the f_{ion} is very small, and therefore, the effects in 1) and 3) are nearly negligible. For example, assuming L-band parameters ($K_a = 600$ Hz/s, $B = 2000$ Hz, $f_0 = 1.26$ GHz, and $V_g = 7124$ m/s) and a linear ionospheric phase component coefficient of $f_{ion} = 1$ Hz (corresponding to a $\Delta\text{TEC} = 1.56$ TECU within the synthetic aperture, where $1 \text{ TECU} = 1 \times 10^{16} \text{ m}^{-2}$), the peak position shift in 2) is $f_{ion}/K_a \times V_g = 11.87$ m. This is already a large shift. The resulting phase error at the peak position solely due to the shifted Doppler centroid frequency is $\pi f_{ion}^2/K_a = \pi/600$ rad = 0.005 rad, which is negligible. Therefore, we will ignore the effects in 1) and 3) in the following analysis.

Note also that the zero-phase position coincides with the zero Doppler time. The phase at the peak position is

$$2\pi\left(f_{\eta_c} + \frac{f_{ion}}{2}\right) \frac{f_{ion}}{K_a} \approx 2\pi f_{\eta_c} \frac{f_{ion}}{K_a} = -2\pi \eta_c f_{ion}. \quad (16)$$

It results from expanding $\text{TEC}(\eta)$ about η_c , instead of η_0 in (6). The phase difference due to the linear ionospheric phase component between the two positions is equivalent to (16). Assuming $f_{\eta_c} = 100$ Hz, the phase at the peak position is $2\pi/6$ rad = 1.05 rad.

C. Effects of Quadratic Ionospheric Phase Component

The quadratic ionospheric phase component, as expressed in (8), alters the linear FM rate of the azimuth signal. This effect is similar to the FM rate mismatch analyzed in previous studies [26]. However, they are not exactly equivalent. Here, the vertex of the quadratic ionospheric phase component is at the Doppler centroid time η_c , whereas, in the case of an FM rate mismatch, the vertex of the quadratic phase error (QPE) is at the zero Doppler time η_0 . Therefore, we need to rederive

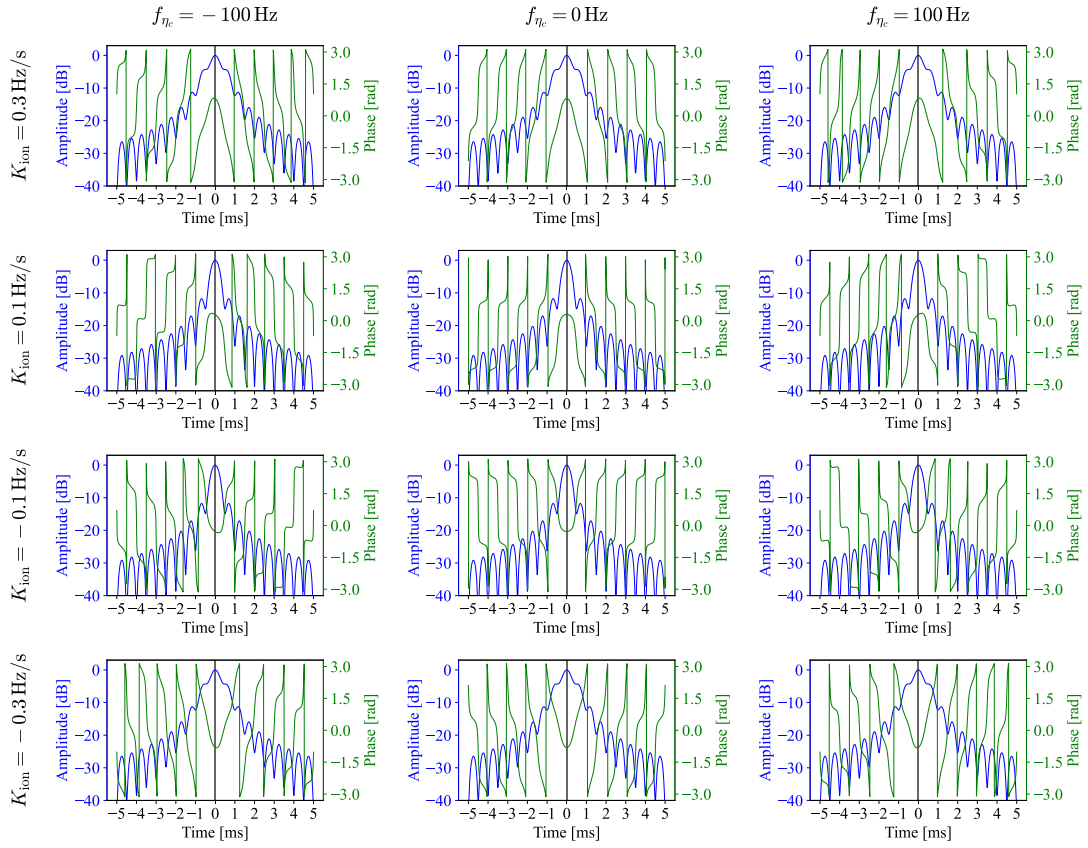


Fig. 2. Focusing results in the presence of a quadratic ionospheric phase component. Here, K_a is 600 Hz/s and B is 2000 Hz. The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

the effects here. Moreover, we will use the Fresnel integrals to obtain a precise result.

With the quadratic ionospheric phase component, the azimuth SAR signal becomes

$$s_{az}^{(2)}(\eta) = A_0 w_a(\eta - \eta_c) \exp\{-j\pi K_a \eta^2\} \times \exp\{j\pi K_{ion}(\eta - \eta_c)^2\}. \quad (17)$$

By applying POSP, the signal spectrum can be derived as

$$\begin{aligned} S_{az}^{(2)}(f_\eta) &\approx A_0 W_a(f_\eta - f_{\eta_c}) \\ &\times \exp\left\{j\pi \left[\frac{f_\eta^2}{K_a - K_{ion}} - 2f_\eta \frac{K_{ion}f_{\eta_c}}{K_a(K_a - K_{ion})} + \frac{K_{ion}}{K_a^2} f_{\eta_c}^2 \right] \right\} \\ &= A_0 W_a(f_\eta - f_{\eta_c}) \exp\left\{j\pi \frac{f_\eta^2}{K_a - K_{ion}}\right\} \\ &\times \exp\left\{-j2\pi f_\eta \frac{K_{ion}f_{\eta_c}}{K_a^2}\right\} \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_{\eta_c}^2\right\}. \end{aligned} \quad (18)$$

Here, the constant term resulting from POSP is omitted. The matched filter, which does not account for ionospheric effects, is identical to the one defined in (11).

After applying this matched filter, the spectrum is given by

$$\begin{aligned} S_{az,focused}^{(2)}(f_\eta) &= S_{az}^{(2)}(f_\eta) \cdot H(f_\eta) \\ &= A_0 W_a(f_\eta - f_{\eta_c}) \\ &\times \exp\left\{j\pi f_\eta^2 \left(\frac{1}{K_a - K_{ion}} - \frac{1}{K_a} \right)\right\} \\ &\times \exp\left\{-j2\pi f_\eta \frac{K_{ion}f_{\eta_c}}{K_a^2}\right\} \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_{\eta_c}^2\right\} \\ &\approx A_0 W_a(f_\eta - f_{\eta_c}) \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_{\eta_c}^2\right\} \\ &\times \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_\eta^2\right\} \exp\left\{-j2\pi f_\eta \frac{K_{ion}f_{\eta_c}}{K_a^2}\right\}. \end{aligned} \quad (19)$$

We define a signal consisting solely of the quadratic ionospheric phase component as

$$S_{quadratic}(f_\eta) = W_a(f_\eta) \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_\eta^2\right\}. \quad (20)$$

Its corresponding time-domain signal is then obtained through an inverse Fourier transform

$$\begin{aligned} s_{quadratic}(\eta) &= \int_{-\infty}^{\infty} S_{quadratic}(f_\eta) \exp\{j2\pi f_\eta \eta\} df_\eta \\ &= \int_{-\infty}^{\infty} W_a(f_\eta) \exp\left\{j\pi \frac{K_{ion}}{K_a^2} f_\eta^2\right\} \exp\{j2\pi f_\eta \eta\} df_\eta \end{aligned}$$

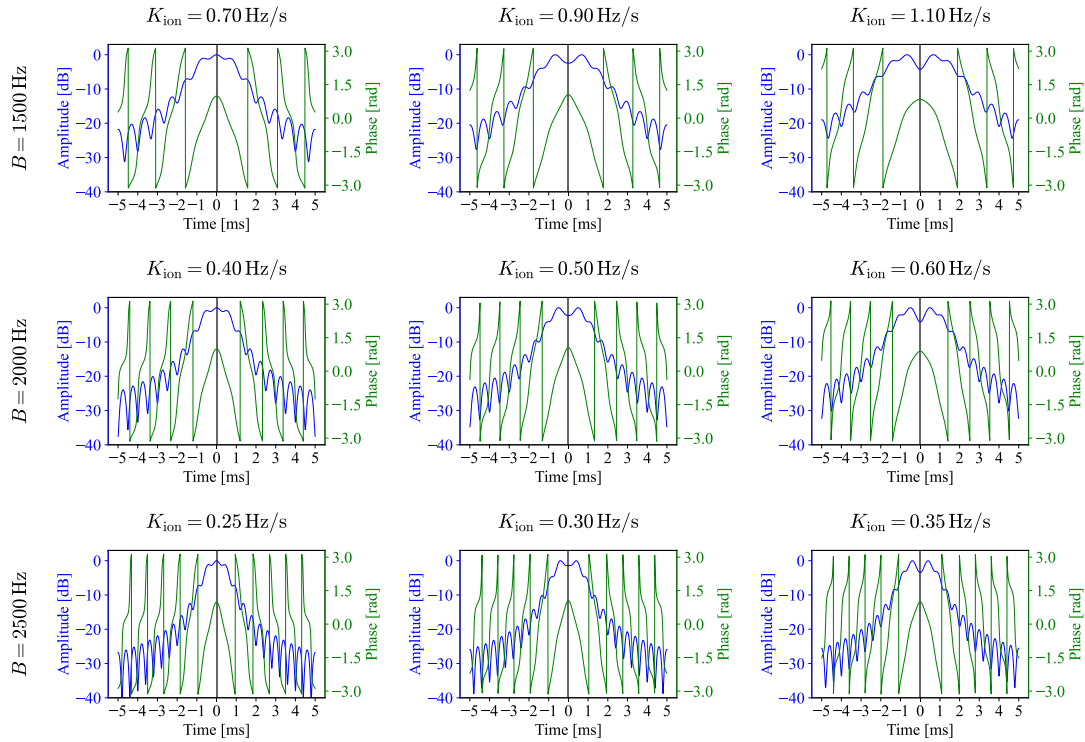


Fig. 3. Splitting of the main lobe when K_{ion} exceeds a certain threshold. The threshold depends on the azimuth bandwidth. Here, K_a is 600 Hz/s and f_{η_c} is 0 Hz. The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the axis of symmetry of the amplitude curve.

$$\times \exp \{j2\pi f_{\eta} \eta\} df_{\eta}. \quad (21)$$

With $s_{\text{quadratic}}(\eta)$, the focused time-domain signal $s_{\text{az,focused}}^{(2)}(\eta)$, obtained by applying an inverse Fourier transform to $S_{\text{az,focused}}^{(2)}(f_{\eta})$ in (19), can be expressed as

$$\begin{aligned} s_{\text{az,focused}}^{(2)}(\eta) &= \int_{-\infty}^{\infty} S_{\text{az,focused}}^{(2)}(f_{\eta}) \exp \{j2\pi f_{\eta} \eta\} df_{\eta} \\ &= \int_{-\infty}^{\infty} A_0 W_a(f_{\eta} - f_{\eta_c}) \exp \left\{ j\pi \frac{K_{\text{ion}}}{K_a^2} f_{\eta}^2 \right\} \\ &\quad \times \exp \left\{ -j2\pi f_{\eta} \frac{K_{\text{ion}} f_{\eta_c}}{K_a^2} \right\} \exp \left\{ j\pi \frac{K_{\text{ion}}}{K_a^2} f_{\eta_c}^2 \right\} \\ &\quad \times \exp \{j2\pi f_{\eta} \eta\} df_{\eta} \\ &= A_0 \int_{-\infty}^{\infty} W_a(f_{\eta} - f_{\eta_c}) \exp \left\{ j\pi \frac{K_{\text{ion}}}{K_a^2} (f_{\eta} - f_{\eta_c})^2 \right\} \\ &\quad \times \exp \{j2\pi f_{\eta} \eta\} df_{\eta} \\ &= A_0 \int_{-\infty}^{\infty} S_{\text{quadratic}}(f_{\eta} - f_{\eta_c}) \exp \{j2\pi f_{\eta} \eta\} df_{\eta} \\ &= A_0 s_{\text{quadratic}}(\eta) \exp \{j2\pi f_{\eta_c} \eta\}. \end{aligned} \quad (22)$$

The remaining task is to solve for $s_{\text{quadratic}}(\eta)$. We provide the details of our solution in Appendix C. The Fresnel integrals in (101), in Appendix C shown at the bottom of the page 27, can be accurately evaluated [28], [29]. With $s_{\text{quadratic}}(\eta)$, $s_{\text{az,focused}}^{(2)}(\eta)$ can be subsequently obtained.

As a result, the target is focused with a phase error arising from $s_{\text{quadratic}}(\eta)$. To further examine the characteristics of the focusing result, we vary the quadratic ionospheric phase component coefficient K_{ion} , the Doppler centroid frequency f_{η_c} , and the azimuth bandwidth B . The resulting $s_{\text{az,focused}}^{(2)}(\eta)$ for a target with unit reflectivity is shown in Figs. 2 and 3.

As expected, the phase of the target depends on K_{ion} . As K_{ion} increases, the main lobe broadens, and the sidelobes merge into the main lobe. When K_{ion} exceeds a certain threshold, which decreases with increasing azimuth bandwidth, the main lobe splits into two distinct peaks, as illustrated in Fig. 3. In this case, accurately characterizing the focused signal becomes difficult; hence, we consider only the case where K_{ion} does not exceed this threshold. The peak position of the focusing result cannot be directly derived from the expression of $s_{\text{quadratic}}(\eta)$. However, as illustrated in Fig. 2, the peak position remains unaffected by the quadratic ionospheric phase component and is still at the zero Doppler position. This observation will be further validated by the analytical derivations in Section II-D. In contrast, the FM rate error shifts the target peak position, as derived in [26].

A closed-form approximation for the phase of $s_{\text{az,focused}}^{(2)}(\eta)$, denoted by $\Phi_{\text{az,focused}}^{(2)}(\eta)$, can be derived using the result in Appendix D. By combining (107) in Appendix D and (22), $\Phi_{\text{az,focused}}^{(2)}(\eta)$ can be approximated as

$$\begin{aligned} \Phi_{\text{az,focused}}^{(2)}(\eta) &= \Phi_{\text{quadratic}}(\eta) + 2\pi f_{\eta_c} \eta \\ &\approx -\pi \frac{K_a^2}{K_{\text{ion}}} \eta^2 \pm \frac{\pi}{6} \frac{[g_H(\eta)^3 - g_L(\eta)^3]}{g_H(\eta) - g_L(\eta)} \\ &\quad + 2\pi f_{\eta_c} \eta \end{aligned} \quad (23)$$

where $g_L(\eta) = (2|K_{\text{ion}}|/K_a^2)^{1/2}(-B/2 + \eta K_a^2/K_{\text{ion}})$, and $g_H(\eta) = (2|K_{\text{ion}}|/K_a^2)^{1/2}(B/2 + \eta K_a^2/K_{\text{ion}})$. At the peak position ($\eta = 0$), it is given by

$$\Phi_{\text{az,focused}}^{(2)}(\eta = 0) \approx \pm \frac{\pi}{6} \frac{[g_H(0)^3 - g_L(0)^3]}{g_H(0) - g_L(0)}$$

$$= \frac{\pi}{3} \frac{K_{\text{ion}}}{K_a^2} \left(\frac{B}{2} \right)^2. \quad (24)$$

It equals one-third of the QPE derived in [26], where QPE = $\pi \Delta K_a (B/2)^2 / K_a^2$, and ΔK_a is equivalent to K_{ion} used here.

D. Effects of Higher Order Ionospheric Phase Components

Here, we present a generic analysis of the higher order ionospheric phase components under the assumption that their coefficients are sufficiently small. We denote the n th order coefficient ($n \geq 3$) as $\kappa_{\text{ion}}^{(n)}$, and its corresponding ionospheric phase component as $\phi_{\text{ion}}^{(n)}$. From (7) and (8)

$$\begin{cases} \kappa_{\text{ion}}^{(n)} = \frac{1}{n!} \frac{4K}{c f_0} \frac{\partial^n \text{TEC}}{\partial \eta^n} \Big|_{\eta=\eta_c} \\ \phi_{\text{ion}}^{(n)} = \pi \kappa_{\text{ion}}^{(n)} (\eta - \eta_c)^n. \end{cases} \quad (25)$$

Considering only the n th ionospheric phase component, the azimuth SAR signal is given by

$$\begin{aligned} s_{\text{az}}^{(n)}(\eta) &= w_a(\eta - \eta_c) \exp \{-j\pi K_a \eta^2\} \\ &\quad \times \exp \{j\pi \kappa_{\text{ion}}^{(n)} (\eta - \eta_c)^n\}. \end{aligned} \quad (26)$$

By applying POSP, we can approximate the spectrum as

$$\begin{aligned} S_{\text{az}}^{(n)}(f_\eta) &\approx W_a(f_\eta - f_{\eta_c}) \\ &\quad \times \exp \left\{ j\pi \frac{f_\eta^2}{K_a} \right\} \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{(f_\eta - f_{\eta_c})^n}{K_a^n} \right\}. \end{aligned} \quad (27)$$

Here, the constant term resulting from POSP is omitted. After applying the matched filter, $H(f_\eta)$, defined in (11), the spectrum becomes

$$\begin{aligned} S_{\text{az,focused}}^{(n)}(f_\eta) &= S_{\text{az}}(f_\eta) \cdot H(f_\eta) \\ &= W_a(f_\eta - f_{\eta_c}) \\ &\quad \times \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{(f_\eta - f_{\eta_c})^n}{K_a^n} \right\}. \end{aligned} \quad (28)$$

The focused signal is then obtained via an inverse Fourier transform of $S_{\text{az,focused}}^{(n)}(f_\eta)$

$$\begin{aligned} s_{\text{az,focused}}^{(n)}(\eta) &= \int_{-\infty}^{\infty} W_a(f_\eta - f_{\eta_c}) \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{(f_\eta - f_{\eta_c})^n}{K_a^n} \right\} \\ &\quad \times \exp \{j2\pi f_\eta \eta\} df_\eta \\ &\approx \int_{-\infty}^{\infty} \text{rect} \left(\frac{f_\eta - f_{\eta_c}}{B} \right) \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{(f_\eta - f_{\eta_c})^n}{K_a^n} \right\} \\ &\quad \times \exp \{j2\pi f_\eta \eta\} df_\eta \\ &= \exp \{j2\pi f_{\eta_c} \eta\} \int_{-\infty}^{\infty} \text{rect} \left(\frac{f_\eta}{B} \right) \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{f_\eta^n}{K_a^n} \right\} \\ &\quad \times \exp \{j2\pi f_\eta \eta\} df_\eta \end{aligned} \quad (29)$$

where the azimuth envelope is approximated by a rectangular function, and B is the azimuth bandwidth.

Due to the intractability of a direct evaluation of the above integral, we employ a Taylor series expansion to derive an

approximate analytical expression under the assumption that $\kappa_{\text{ion}}^{(n)}$ is sufficiently small. By defining $\epsilon = (-1)^n \kappa_{\text{ion}}^{(n)} / K_a^n$, $f_H = B/2$, $f_L = -B/2$, the integral can be expressed as

$$\begin{aligned} s_n(\eta) &= \int_{-\infty}^{\infty} \text{rect} \left(\frac{f_\eta}{B} \right) \exp \left\{ (-1)^n j\pi \kappa_{\text{ion}}^{(n)} \frac{f_\eta^n}{K_a^n} \right\} \exp \{j2\pi f_\eta \eta\} df_\eta \\ &= \int_{f_L}^{f_H} \exp \{j\pi \epsilon f_\eta^n\} \exp \{j2\pi f_\eta \eta\} df_\eta \\ &\approx \int_{f_L}^{f_H} (1 + j\pi \epsilon f_\eta^n) \exp \{j2\pi f_\eta \eta\} df_\eta \\ &= \int_{f_L}^{f_H} \exp \{j2\pi f_\eta \eta\} df_\eta + j\pi \epsilon \int_{f_L}^{f_H} f_\eta^n \exp \{j2\pi f_\eta \eta\} df_\eta. \end{aligned} \quad (30)$$

The derivation in the third line employs a first-order Taylor expansion about $f_\eta = 0$, given by

$$\begin{aligned} \exp \{j\pi \epsilon f_\eta^n\} &= 1 + j\pi \epsilon f_\eta^n + \frac{(j\pi \epsilon f_\eta^n)^2}{2!} + \dots \\ &\approx 1 + j\pi \epsilon f_\eta^n. \end{aligned} \quad (31)$$

For this approximation to remain valid, the condition $|\pi \epsilon f_\eta^n| < 1$ should be satisfied. Given that the maximum absolute value of f_η is $B/2$, this requirement translates into the constraint on $\kappa_{\text{ion}}^{(n)}$

$$\left| \kappa_{\text{ion}}^{(n)} \right| < \frac{K_a^n}{\pi} \left(\frac{2}{B} \right)^n. \quad (32)$$

Let $s_1(\eta)$ and $s_2(\eta)$ be the first and second terms of $s_n(\eta)$ in (30), respectively. These two terms are related through the n th order derivative of $s_1(\eta)$

$$\begin{aligned} \frac{d^n}{d\eta^n} s_1(\eta) &= \frac{d^n}{d\eta^n} \left[\int_{f_L}^{f_H} \exp \{j2\pi f_\eta \eta\} df_\eta \right] \\ &= (j2\pi)^n \int_{f_L}^{f_H} f_\eta^n \exp \{j2\pi f_\eta \eta\} df_\eta \\ &= j^{n-1} \frac{2^n \pi^{n-1}}{\epsilon} \left[j\pi \epsilon \int_{f_L}^{f_H} f_\eta^n \exp \{j2\pi f_\eta \eta\} df_\eta \right] \\ &= j^{n-1} \frac{2^n \pi^{n-1}}{\epsilon} s_2(\eta). \end{aligned} \quad (33)$$

Applying a Taylor series expansion of $s_1(\eta)$ about $\eta = 0$ yields

$$\begin{aligned} s_1(\eta) &= \int_{f_L}^{f_H} \exp \{j2\pi f_\eta \eta\} df_\eta \\ &= \text{Bsinc}(\pi B \eta) \\ &= B \left[1 - \frac{(\pi B \eta)^2}{3!} + \frac{(\pi B \eta)^4}{5!} - \frac{(\pi B \eta)^6}{7!} \right. \\ &\quad \left. + \dots + (-1)^k \frac{(\pi B \eta)^{2k}}{(2k+1)!} \right] \end{aligned} \quad (34)$$

which subsequently leads to

$$\begin{aligned} s_2(\eta) &= \frac{\epsilon}{j^{n-1} 2^n \pi^{n-1}} \frac{d^n}{d\eta^n} s_1(\eta) \\ &= \left(-\frac{j}{\pi} \right)^{n-1} \frac{\epsilon}{2^n} \frac{d^n}{d\eta^n} \left\{ B \left[1 - \frac{(\pi B \eta)^2}{3!} + \frac{(\pi B \eta)^4}{5!} \right. \right. \end{aligned}$$

$$\left. -\frac{(\pi B\eta)^6}{7!} + \dots + (-1)^k \frac{(\pi B\eta)^{2k}}{(2k+1)!} \right\}. \quad (35)$$

If n is odd, then

$$\begin{aligned} s_2(\eta) &= \frac{(-1)^{\frac{(n-1)}{2}}}{\pi^{n-1}} \frac{\epsilon}{2^n} B \left[(-1)^{\frac{(n+1)}{2}} \frac{(\pi B)^{n+1}}{(n+2)!} (n+1)! \eta \right. \\ &\quad \left. + (-1)^{\frac{(n+3)}{2}} \frac{(\pi B)^{n+3}}{(n+4)!} \frac{(n+3)!}{3!} \eta^3 + \dots \right] \\ &= \frac{\epsilon}{2^n} \left[-\frac{\pi^2 B^{n+2}}{n+2} \eta + \frac{\pi^4 B^{n+4}}{3!(n+4)} \eta^3 + \dots \right]. \end{aligned} \quad (36)$$

Combining $s_1(\eta)$ and $s_2(\eta)$ yields

$$\begin{aligned} s_n(\eta) &= s_1(\eta) + s_2(\eta) \\ &= B - \frac{\epsilon}{2^n} \frac{\pi^2 B^{n+2}}{(n+2)} \eta - \frac{\pi^2 B^3}{6} \eta^2 \\ &\quad + \frac{\epsilon}{2^n} \frac{\pi^4 B^{n+4}}{3!(n+4)} \eta^3 + \dots \\ &\approx B - \frac{\epsilon}{2^n} \frac{\pi^2 B^{n+2}}{(n+2)} \eta - \frac{\pi^2 B^3}{6} \eta^2 \\ &= B + \frac{\kappa_{\text{ion}}^{(n)}}{K_a^n} \frac{\pi^2 B^{n+2}}{2^n(n+2)} \eta - \frac{\pi^2 B^3}{6} \eta^2. \end{aligned} \quad (37)$$

Here, the higher order terms are truncated. Since (37) is purely real, its phase is zero. The peak position of the focused target occurs near the zero Doppler time $\eta = 0$. By setting the first-order derivative of $s_n(\eta)$ to zero

$$\frac{d}{d\eta} s_n(\eta) = \frac{\kappa_{\text{ion}}^{(n)}}{K_a^n} \frac{\pi^2 B^{n+2}}{2^n(n+2)} - \frac{\pi^2 B^3}{3} \eta = 0 \quad (38)$$

we can obtain the peak position

$$\eta = \frac{3}{2^n} \frac{\kappa_{\text{ion}}^{(n)}}{K_a^n} \frac{B^{n-1}}{(n+2)}. \quad (39)$$

These results indicate that an odd-order ionospheric phase component induces a shift in the peak position of the focused target, without changing its phase.

If n is even, $s_n(\eta)$ can be expressed as

$$\begin{aligned} s_n(\eta) &= s_1(\eta) + s_2(\eta) \\ &= B \text{sinc}(\pi B\eta) + \left(-\frac{j}{\pi} \right)^{n-1} \frac{\epsilon B}{2^n} \frac{d^n}{d\eta^n} \text{sinc}(\pi B\eta) \\ &= B \text{sinc}(\pi B\eta) + (-1)^{\frac{n}{2}} j \frac{\epsilon B}{2^n \pi^{n-1}} \frac{d^n}{d\eta^n} \text{sinc}(\pi B\eta). \end{aligned} \quad (40)$$

Since $\text{sinc}(\pi B\eta)$ and its even-order derivative $d^n[\text{sinc}(\pi B\eta)]/d\eta^n$ are even functions, it follows that:

$$\begin{aligned} s_n(-\eta) &= B \text{sinc}(-\pi B\eta) \\ &\quad + (-1)^{\frac{n}{2}} j \frac{\epsilon B}{2^n \pi^{n-1}} \frac{d^n}{d\eta^n} \text{sinc}(-\pi B\eta) \\ &= B \text{sinc}(\pi B\eta) \\ &\quad + (-1)^{\frac{n}{2}} j \frac{\epsilon B}{2^n \pi^{n-1}} \frac{d^n}{d\eta^n} \text{sinc}(\pi B\eta) \\ &= s_n(\eta). \end{aligned} \quad (41)$$

This identity demonstrates that $s_n(\eta)$ is symmetric about $\eta = 0$.

$s_n(\eta)$ can be approximated around $\eta = 0$ using the Taylor series expansion in (34) and retaining terms up to second order

$$\begin{aligned} s_n(\eta) &= s_1(\eta) + s_2(\eta) \\ &= \left(B - \frac{\pi^2 B^3}{3!} \eta^2 + \dots \right) \\ &\quad + (-1)^{\frac{n}{2}} j \frac{\epsilon B}{2^n \pi^{n-1}} \left[(-1)^{\frac{n}{2}} \frac{(\pi B)^n}{(n+1)!} n! \right. \\ &\quad \left. + (-1)^{\frac{(n+2)}{2}} \frac{(\pi B)^{n+2}}{(n+3)!} \frac{(n+2)!}{2!} \eta^2 + \dots \right] \\ &\approx \left(B - \frac{\pi^2 B^3}{3!} \eta^2 \right) \\ &\quad + j \frac{\epsilon}{2^n} \left[\frac{\pi B^{n+1}}{(n+1)} - \frac{\pi^3 B^{n+3}}{2(n+3)} \eta^2 \right]. \end{aligned} \quad (42)$$

Its amplitude is given by

$$\begin{aligned} A_n(\eta) &= \sqrt{\left(B - \frac{\pi^2 B^3}{3!} \eta^2 \right)^2 + \frac{\epsilon^2}{2^{2n}} \left[\frac{\pi B^{n+1}}{(n+1)} - \frac{\pi^3 B^{n+3}}{2(n+3)} \eta^2 \right]^2}. \end{aligned} \quad (43)$$

The amplitude function attains its maximum at $\eta = 0$, which is the zero Doppler time.

The phase of the focused target is given by

$$\begin{aligned} \Phi_n(\eta) &= \arctan \left\{ \frac{\frac{\epsilon}{2^n} \left[\frac{\pi B^{n+1}}{(n+1)} - \frac{\pi^3 B^{n+3}}{2(n+3)} \eta^2 \right]}{B - \frac{\pi^2 B^3}{3!} \eta^2} \right\} \\ &\approx \frac{\frac{\epsilon}{2^n} \left[\frac{\pi B^{n+1}}{(n+1)} - \frac{\pi^3 B^{n+3}}{2(n+3)} \eta^2 \right]}{B - \frac{\pi^2 B^3}{3!} \eta^2}. \end{aligned} \quad (44)$$

Specifically, the phase at the peak position is

$$\begin{aligned} \Phi_n(\eta = 0) &\approx \frac{\epsilon}{2^n} \frac{\pi B^n}{n+1} \\ &= \frac{\pi \kappa_{\text{ion}}^{(n)}}{2^n(n+1)} \left(\frac{B}{K_a} \right)^n. \end{aligned} \quad (45)$$

Equation (45) indicates that $\Phi_n(\eta = 0)$ increases with $\kappa_{\text{ion}}^{(n)}$ and B/K_a . Therefore, in contrast to the odd-order ionospheric phase components, the even-order ionospheric phase components do not induce a shift in the peak position of the focused target, but instead induce a phase error.

Here, we consider the cubic and quartic ionospheric phase components as examples. The cubic ionospheric phase component with coefficient δ_{ion} , according to (39), induces a shift in the peak position of the focused target by

$$\eta_{\text{cubic}} = \frac{3}{2^n} \frac{\delta_{\text{ion}}}{K_a^n} \frac{B^{n-1}}{(n+2)} \Big|_{n=3} = \frac{3\delta_{\text{ion}}}{40} \frac{B^2}{K_a^3} \quad (46)$$

but does not change its phase. Conversely, according to (45), the quartic ionospheric phase component with coefficient ϵ_{ion} induces a phase error at the peak position of the focused target

$$\Phi_{\text{quartic}} = \frac{\pi \epsilon_{\text{ion}}}{2^n(n+1)} \left(\frac{B}{K_a} \right)^n \Big|_{n=4} = \frac{\pi \epsilon_{\text{ion}}}{80} \left(\frac{B}{K_a} \right)^4 \quad (47)$$

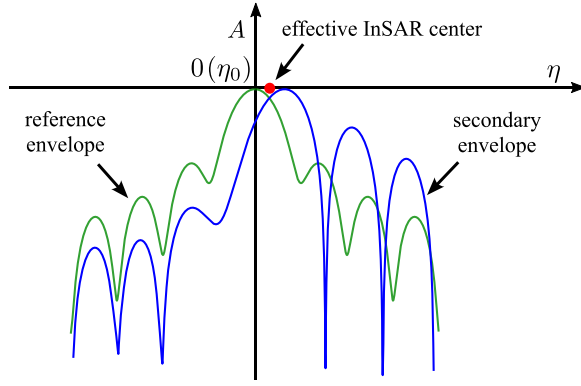


Fig. 4. Effective InSAR center.

but does not shift its peak position. According to (32), the following conditions must be satisfied for these results to remain valid:

$$\begin{cases} |\delta_{\text{ion}}| < \frac{K_a^3}{\pi} \left(\frac{2}{B}\right)^3 \\ |\varepsilon_{\text{ion}}| < \frac{K_a^4}{\pi} \left(\frac{2}{B}\right)^4 \end{cases} \quad (48)$$

E. Summary

The following summarizes the individual effects of different ionospheric phase components on the focused target.

- 1) The odd-order ionospheric phase components (e.g., 1st, 3rd, 5th,...) induce a shift in the peak position of the focused target, but do not change its phase.
- 2) The even-order ionospheric phase components (e.g., 2nd, 4th, 6th,...) induce an error in its phase, but do not shift its peak position. The phase error is an even function that is symmetric about the peak position. In the absence of the peak position shift induced by odd-order ionospheric phase components, the phase error is symmetric about the zero Doppler position ($\eta = 0$).
- 3) The 2nd-order and higher order ionospheric phase components also result in defocusing of the target.

Moreover, the linear phase induced by the Doppler centroid frequency is also essential. The linear phase is an odd function that is antisymmetric about the zero Doppler position ($\eta = 0$). We further emphasize that the phase error is evaluated by comparing the signals at identical azimuth positions with respect to the same origin, even if the peak position is shifted. The coupling effects among different components, however, are complex and should be further investigated in future work.

The distinct effects of the odd- and even-order ionospheric phase components can be interpreted as follows. The odd-order ionospheric phase components are odd functions and antisymmetric about the Doppler centroid time. This antisymmetry effectively applies a differential “push” to the waveform, shifting the peak position. In contrast, the even-order ionospheric phase components are even functions, symmetric about the Doppler centroid time. They

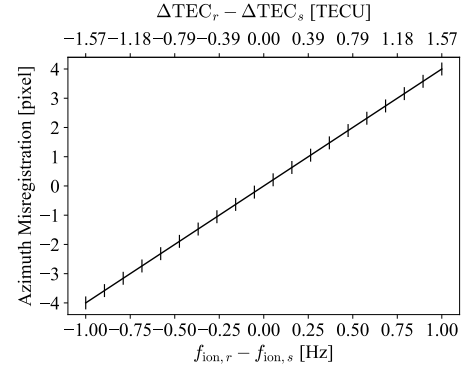


Fig. 5. Azimuth misregistration induced by the linear ionospheric phase component. K_a is 600 Hz/s, B is 2000 Hz, PRF is 2400 Hz, and f_0 is 1.26 GHz.

cause symmetric broadening, compression, or distortion of the point target’s impulse response without shifting its peak position.

The derived effects are valid in most real-world scenarios. To demonstrate this validity, we assume the parameters of an L-band satellite SAR system ($B = 2000$ Hz, $K_a = 600$ Hz/s, PRF = 2400 Hz, $f_0 = 1.26$ GHz, and $V_g = 7124$ m/s). For the linear ionospheric phase component, the peak position shift in (14) holds for any f_{ion} . A $f_{\text{ion}} = 1$ Hz (corresponding to a $\Delta\text{TEC} = 1.56$ TECU within the synthetic aperture) leads to a peak position shift of approximately four azimuth pixels (approximately 11.87 m), representing a significant impact in real-world scenarios. The coefficients of the quadratic and higher order ionospheric phase components are expected to be less than this f_{ion} value in most real-world scenarios, as the coefficients of the ionospheric phase components generally decrease with increasing order. For the coefficient K_{ion} of the quadratic ionospheric phase component, the main lobe splits into two distinct peaks when K_{ion} exceeds a certain threshold, which is approximately $|K_{\text{ion}}| = 0.4$ Hz/s, as shown in Fig. 3. Therefore, the derived phase error in (23) and (24) is valid for $|K_{\text{ion}}| < 0.4$ Hz/s. According to (24), a $K_{\text{ion}} = 0.36$ Hz/s leads to a phase error of approximately $\pi/3$ rad at the peak position. For the cubic (δ_{ion}) and quartic (ε_{ion}) ionospheric phase components, the peak position shift given in (46) and the phase error given in (47) are valid when $|\delta_{\text{ion}}| < 0.069$ Hz/s² and $|\varepsilon_{\text{ion}}| < 0.041$ Hz/s³ according to (48). The simulation results in Sections IV-A and IV-B show that these requirements can actually be relaxed to $|\delta_{\text{ion}}| < 0.4$ Hz/s² and $|\varepsilon_{\text{ion}}| < 0.1$ Hz/s³. A $\delta_{\text{ion}} = 0.30$ Hz/s² can shift the peak position by approximately one pixel, and an $\varepsilon_{\text{ion}} = 0.10$ Hz/s³ can induce a phase error of nearly 0.48 rad at the peak position.

III. EFFECTS ON INSAR AND CLOSURE PHASES

This section examines the effects of along-track ionospheric variations within the synthetic aperture on InSAR, in which the two SAR images experience spatially or temporally distinct ionospheric conditions.

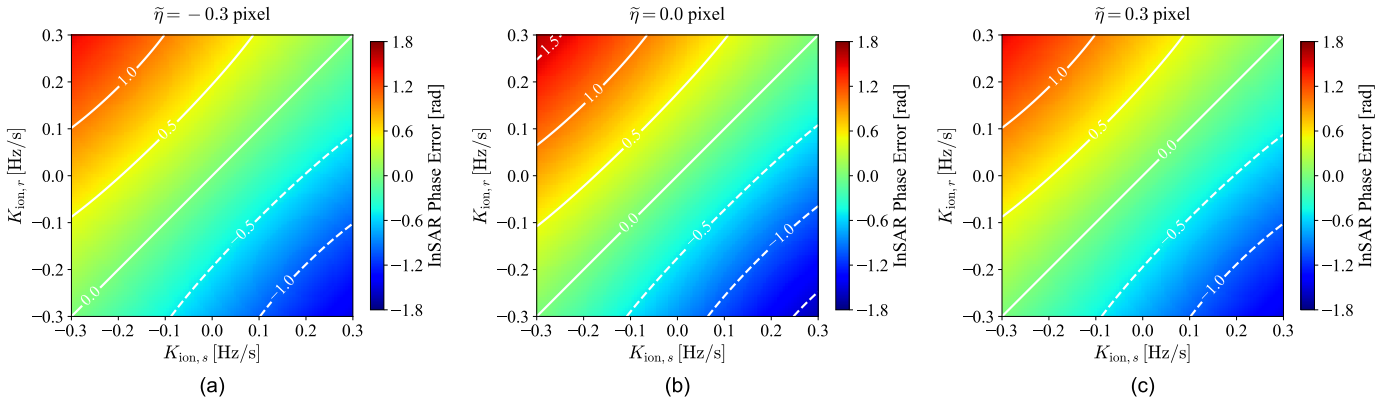


Fig. 6. InSAR phase error induced by the quadratic ionospheric phase component. K_a is 600 Hz/s, B is 2000 Hz, PRF is 2400 Hz, and $\Delta\eta_r = 0$ s. (a) $\tilde{\eta} = -0.3$ pixel. (b) $\tilde{\eta} = 0.0$ pixel. (c) $\tilde{\eta} = 0.3$ pixel. The variation of $\tilde{\eta}$ may arise from the odd-order ionospheric phase components.

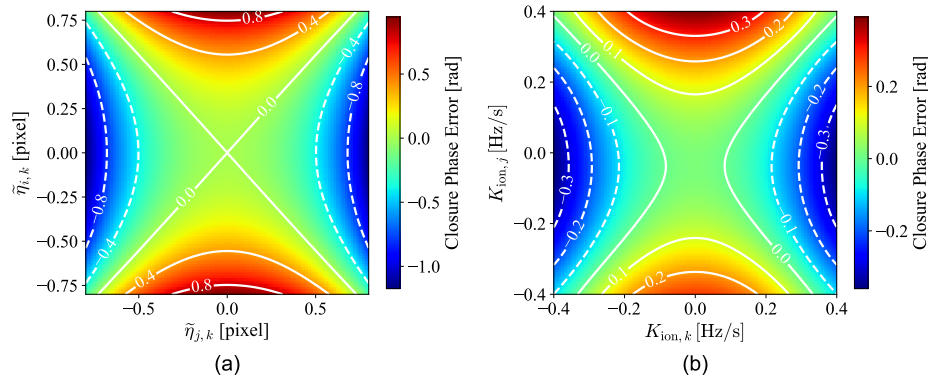


Fig. 7. Closure phase error induced by the quadratic ionospheric phase component. K_a is 600 Hz/s, B is 2000 Hz, and PRF is 2400 Hz. (a) $[K_{ion,i}, K_{ion,j}, K_{ion,k}] = [0.1, 0.2, -0.3]$ Hz/s, and $\tilde{\eta}_{i,j} = 0.0$ pixel. f_{η_c} is 0 Hz. (b) $[\tilde{\eta}_{i,j}, \tilde{\eta}_{i,k}, \tilde{\eta}_{j,k}] = [0.2, 0.3, -0.3]$ pixel, and $K_{ion,i} = 0.1$ Hz/s. f_{η_c} is 100 Hz. The variation of $\tilde{\eta}$ may arise from the odd-order ionospheric phase components.

A. Underlying Mechanism for a Point Target

If the reference and secondary signals from a point target are precisely coregistered, such that their peak positions coincide, the effective InSAR center will also coincide with the peak positions. However, if a misregistration occurs due to the relative azimuth shift and defocusing induced by the ionosphere, as shown in Fig. 4, the effective InSAR center will shift to a position intermediate between the two SAR signal peaks. Accordingly, the InSAR phase corresponds to the phase difference at the effective InSAR center. Moreover, if the positions of the effective InSAR centers differ among the interferograms forming an InSAR phase triplet, a closure phase error can arise.

B. Azimuth Misregistration for a Point Target

Assuming the azimuth offset due to imaging geometry has been precisely calculated, the azimuth misregistrations are caused by the differential shifts in the peak positions between the reference and secondary SAR signals induced by the odd-order ionospheric phase components. These differential shifts can exhibit strong spatial variability across the image, as shown in Section V, and therefore it is challenging to estimate them consistently with high precision and accuracy over the entire image using existing offset estimation meth-

ods, such as cross correlation and spectral-diversity methods [30], [31].

Assuming the total azimuth shifts in the reference and secondary signals are $\Delta\eta_{ion,r}$ and $\Delta\eta_{ion,s}$, respectively, the misregistration is

$$\Delta\eta_{ion} = \Delta\eta_{ion,r} - \Delta\eta_{ion,s}. \quad (49)$$

Using the linear ionospheric phase component as an illustrative case, the misregistration becomes

$$\Delta\eta_{ion} = \Delta\eta_{ion,r} - \Delta\eta_{ion,s} = \frac{f_{ion,r} - f_{ion,s}}{K_a} \quad (50)$$

where $f_{ion,r}$ and $f_{ion,s}$ are the linear ionospheric phase component coefficients of the reference and secondary SAR signals, respectively.

Fig. 5 presents the azimuth misregistrations due to the linear ionospheric phase component, computed under typical system configurations. The results show that a difference of only 0.1 Hz in the linear ionospheric phase component coefficients (corresponding to a ΔTEC difference of 0.16 TECU within the synthetic aperture) can induce a misregistration of 0.4 azimuth pixels when K_a is 600 Hz/s, B is 2000 Hz, PRF is 2400 Hz, and f_0 is 1.26 GHz.

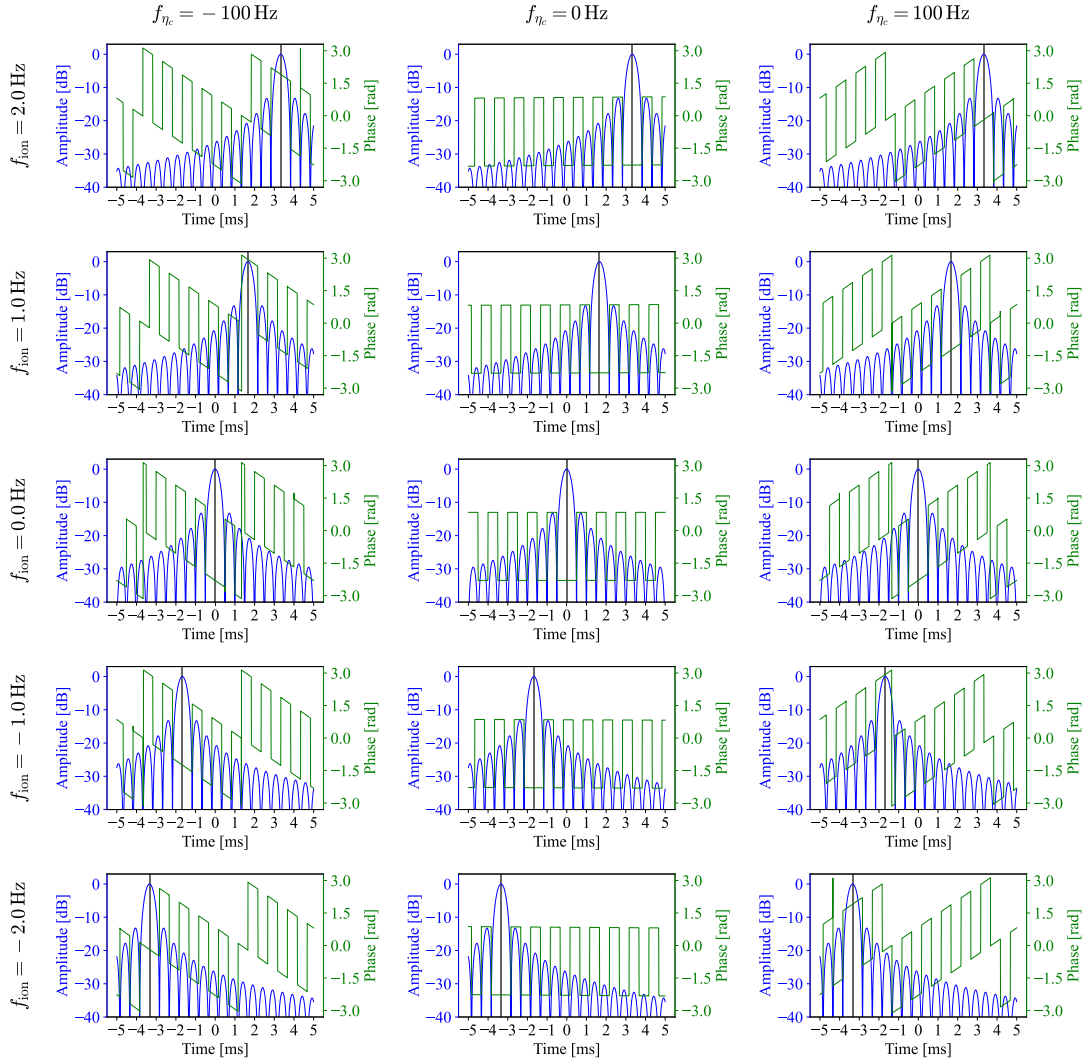


Fig. 8. Focused target with different linear ionospheric phase components ($f_{\text{ion}} = 0, \pm 1.0, \pm 2.0$ Hz) under different Doppler centroid frequencies ($f_{\eta_c} = 0, \pm 100$ Hz). The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

C. InSAR Phase Error for a Point Target

It is initially assumed that the secondary signal is coregistered to the reference signal by aligning their zero Doppler positions using geometric offsets. Note that we have defined the origin of the azimuth time as the zero Doppler time throughout this article. The origins of the reference and secondary signals then coincide. Assume that the effective InSAR center is located at $\tilde{\eta}$, which is identical for the reference signal, the secondary signal, and the interferogram. Considering the peak position shifts induced by the odd-order ionospheric phase components, an additional offset $\Delta\eta_r$ estimated via, e.g., cross correlation, may be applied. Consequently, the corresponding effective InSAR center in the secondary signal becomes $\tilde{\eta} + \Delta\eta_r$.

Assume that $s_{\text{az,focused},r}(\eta)$ and $s_{\text{az,focused},s}(\eta)$ are the focused reference and secondary signals, and $i(\eta)$ is the resulting interferogram. Although we are analyzing a point target, multilooking is performed, and we use an expectation operator $E(\cdot)$ to represent the resulting interferogram after multilooking.

The phase of the multilooked interferogram is then

$$\arg \{E[i(\eta)]\} = \arg \{E[s_{\text{az,focused},r}(\eta) s_{\text{az,focused},s}^*(\eta)]\} \approx \arg \{i(\tilde{\eta})\} \quad (51)$$

where we have used the interferometric phase at the effective InSAR center $\tilde{\eta}$ to approximate $\arg\{E[i(\eta)]\}$.

The phase error in the reference signal at its effective InSAR center arises from the linear phase induced by the Doppler centroid frequency and the phase errors induced by all the even-order ionospheric phase components

$$\Phi_{\text{az,focused},r}(\tilde{\eta}) = 2\pi f_{\eta_c} \tilde{\eta} + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) \quad (52)$$

where $\Phi_{\text{az,focused},r}^{(\text{even})}(\eta)$ represents the phase error induced by all the even-order ionospheric phase components of the reference signal. It is not a simple summation of these phase errors, and its analytical form is difficult to derive. Similarly, the phase error in the secondary signal at its effective InSAR center is

$$\Phi_{\text{az,focused},s}(\tilde{\eta} + \Delta\eta_r) = 2\pi f_{\eta_c} (\tilde{\eta} + \Delta\eta_r) + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r) \quad (53)$$

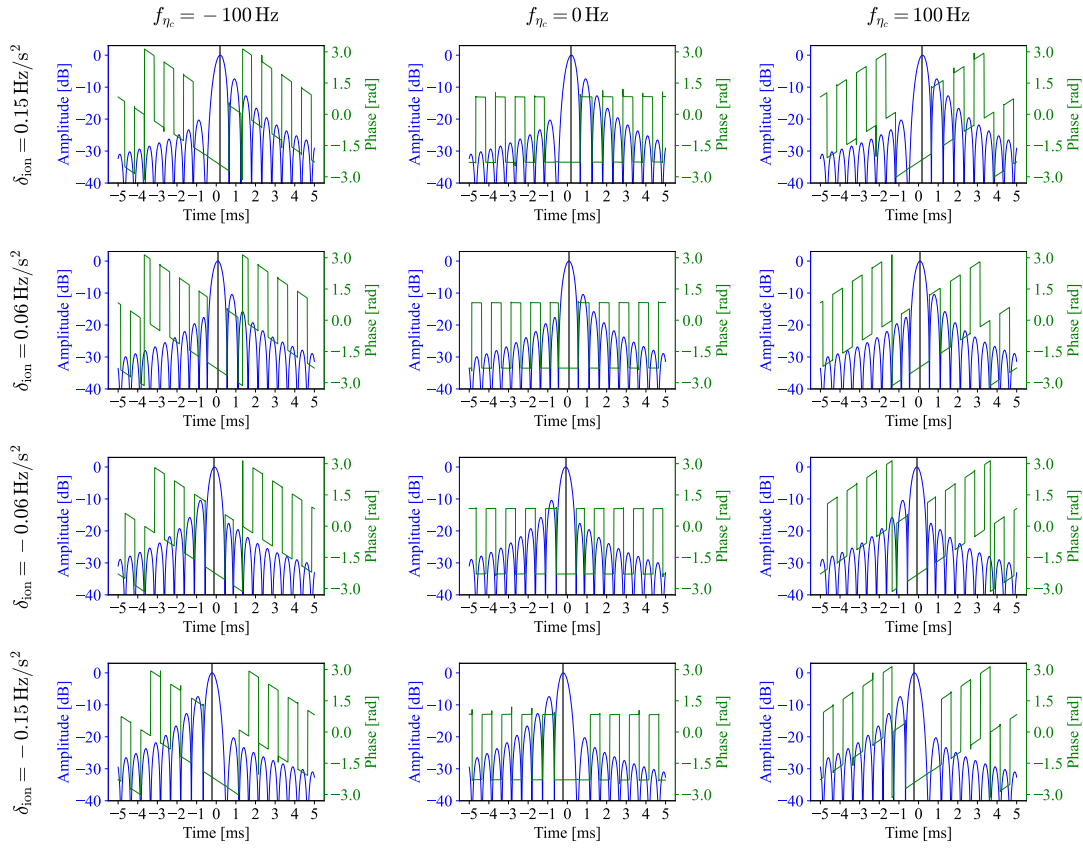


Fig. 9. Focused target with different cubic ionospheric phase components ($\delta_{\text{ion}} = \pm 0.06, \pm 0.15 \text{ Hz/s}^2$) under different Doppler centroid frequencies ($f_{\eta_c} = 0, \pm 100 \text{ Hz}$). The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

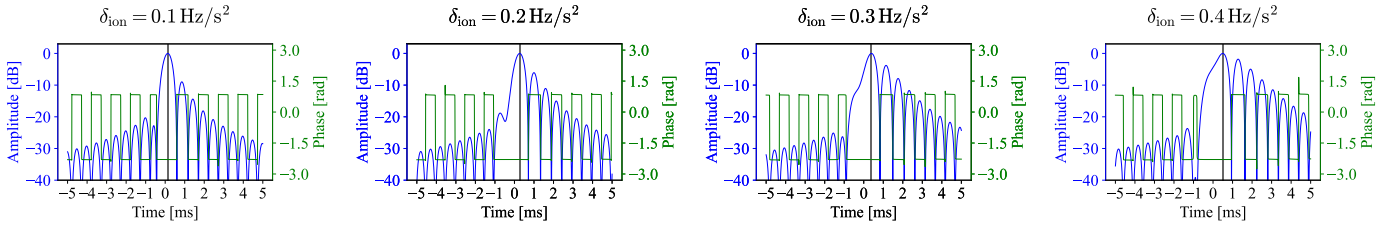


Fig. 10. Focused target with large δ_{ion} values. f_{η_c} is 0 Hz. The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

where $\Phi_{\text{az,focused},s}^{(\text{even})}(\eta)$ represents the phase errors induced by all the even-order ionospheric phase components of the secondary signal. Following (51), the phase error in the multilooked interferogram can be approximated by the phase error in the single-look interferogram at its effective InSAR center

$$\begin{aligned} \Phi_{\text{int}}(\tilde{\eta}) &= \Phi_{\text{az,focused},r}(\tilde{\eta}) - \Phi_{\text{az,focused},s}(\tilde{\eta} + \Delta\eta_r) \\ &= -2\pi f_{\eta_c} \Delta\eta_r + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{aligned} \quad (54)$$

The effective InSAR center of the interferogram is expected to be close to its peak position.

Under typical system configurations and assuming $\Delta\eta_r = 0$, we take the quadratic ionospheric phase component as an example to show the InSAR phase error in Fig. 6. The secondary signal has been coregistered to the reference signal using geometric offsets.

D. Distributed Target

A distributed target can be regarded as a combination of multiple point targets. The ionosphere generally varies smoothly in space. The original radar signals of all individual point targets within a distributed target can be considered to be advanced by the same ionospheric phase, because the spatial variations of the ionosphere are generally negligible within a resolution cell or the size of the distributed target. After focusing, the resulting ionospheric effects are identical for each point target, and consequently, the distributed target as a whole will exhibit effects similar to those observed for a point target, which will be demonstrated below.

For the signals of all point targets within the distributed target, we define a common origin, which is located at the center of the distributed target. It is also the zero Doppler position of the point target at the center of the distributed

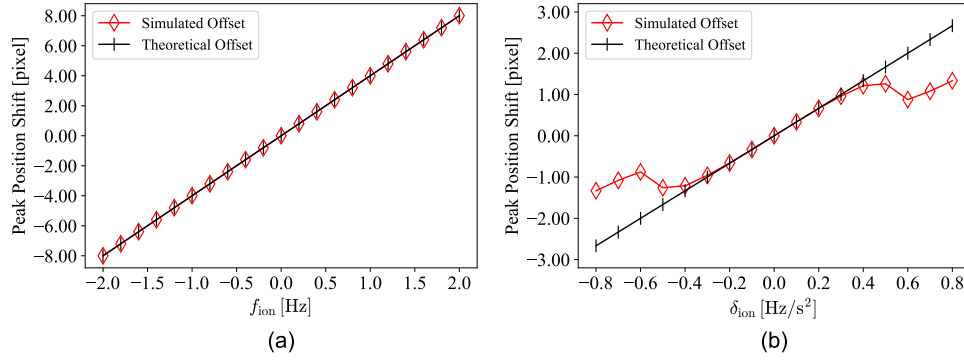


Fig. 11. Shift in the peak position of the focused target in the presence of (a) linear and (b) cubic ionospheric phase components. f_{η_c} is 0 Hz.

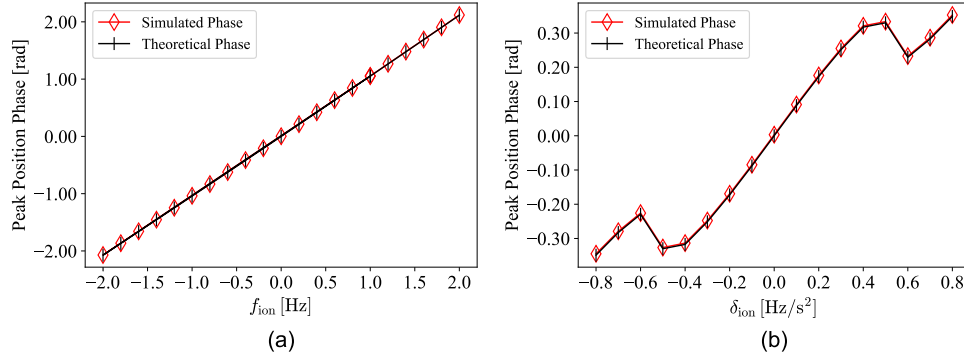


Fig. 12. Phase at the peak position in the presence of (a) linear and (b) cubic ionospheric phase components. f_{η_c} is 100 Hz.

target. Assume that the focused signal for a point target with unit reflectivity at this position is given by

$$s_{az,focused}(\eta) = A_{az,focused}(\eta) \exp \{j\Phi_{az,focused}(\eta)\} \quad (55)$$

where $A_{az,focused}(\eta)$ and $\Phi_{az,focused}(\eta)$ are the amplitude envelope and phase of the signal, respectively. Then, the focused signal for a point target at an arbitrary position can be expressed as

$$\begin{aligned} s_{az,focused,i}(\eta) &= r_i s_{az,focused}(\eta - \eta_i) \\ &= r_i A_{az,focused}(\eta - \eta_i) \\ &\quad \times \exp \{j\Phi_{az,focused}(\eta - \eta_i)\} \end{aligned} \quad (56)$$

where r_i is the complex reflectivity of the point target, and η_i is the position of the target.

The total scattered electromagnetic wave from the distributed target is the superposition of those from all constituent point targets, and the resulting signal of the distributed target is

$$s_{DS}(\eta) = \sum_{i=1}^N r_i A_{az,focused}(\eta - \eta_i) \exp \{j\Phi_{az,focused}(\eta - \eta_i)\} \quad (57)$$

where we have assumed that the point targets are independent.

At the origin ($\eta = 0$), $s_{DS}(\eta)$ becomes

$$s_{DS}(0) = \sum_{i=1}^N r_i A_{az,focused}(-\eta_i) \exp \{j\Phi_{az,focused}(-\eta_i)\}. \quad (58)$$

If the envelope of each point target, $A_{az,focused}(\eta - \eta_i)$, is shifted by the same amount $\Delta\eta$ due to the odd-order ionospheric phase components, the envelope of $s_{DS}(\eta)$ will also be approximately shifted by $\Delta\eta$.

Assume that $s_{az,focused,r,i}(\eta)$ and $s_{az,focused,s,i}(\eta)$ are the focused reference and secondary signals for point target i located at η_i . They can be given by

$$\begin{cases} s_{az,focused,r,i}(\eta) = r_i s_{az,focused,r}(\eta - \eta_i) \\ s_{az,focused,s,i}(\eta) = r_i s_{az,focused,s}(\eta - \eta_i) \end{cases} \quad (59)$$

where $s_{az,focused,r}(\eta)$ and $s_{az,focused,s}(\eta)$ are the focused reference and secondary signals for the point target with unit reflectivity located at the origin, as given in (55). Then, the signals of the distributed target for the two acquisitions, $s_{DS,r}(\eta)$ and $s_{DS,s}(\eta)$, can be expressed as

$$\begin{cases} s_{DS,r}(\eta) = \sum_{i=1}^N s_{az,focused,r,i}(\eta) \\ s_{DS,s}(\eta) = \sum_{i=1}^N s_{az,focused,s,i}(\eta). \end{cases} \quad (60)$$

The expected or multilooked interferogram of the target is

$$\begin{aligned} E[i(\eta)] &= E[s_{DS,r}(\eta) s_{DS,s}^*(\eta)] \\ &= E \left[\sum_{i=1}^N s_{az,focused,r,i}(\eta) \sum_{j=1}^N s_{az,focused,s,j}^*(\eta) \right] \end{aligned}$$

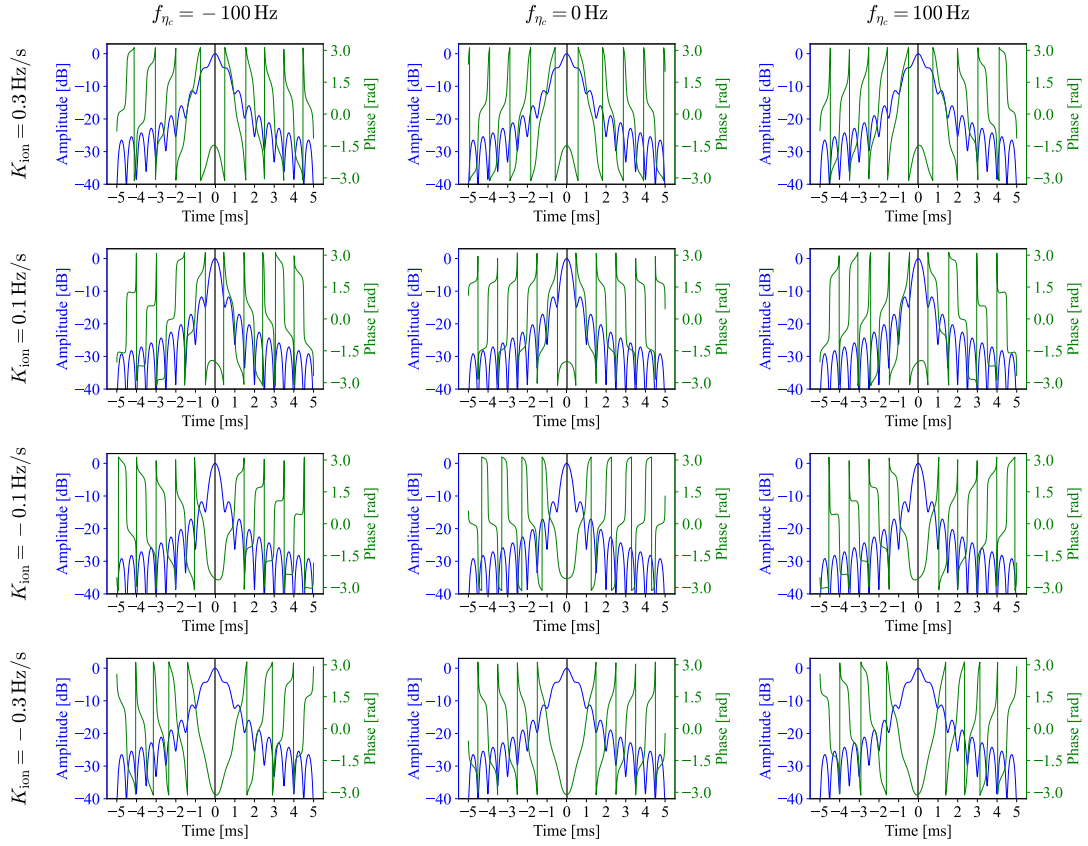


Fig. 13. Focused target with different quadratic ionospheric phase components ($K_{\text{ion}} = \pm 0.1, \pm 0.3$ Hz/s) under different Doppler centroid frequencies ($f_{\eta_c} = 0, \pm 100$ Hz). The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

$$\begin{aligned}
 &= E \left[\sum_{i=1}^N s_{\text{az,focused},r,i}(\eta) s_{\text{az,focused},s,i}^*(\eta) \right] \\
 &= \sum_{i=1}^N E \left[|r_i|^2 \right] s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \\
 &= \sum_{i=1}^N E [\sigma_i] s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \quad (61)
 \end{aligned}$$

where $|r_i|^2 = \sigma_i$ is the radar cross section (RCS) of the point target i (neglecting 4π) in the far field [32]. The cross-product terms between different point targets have canceled out, leaving only the self-interaction terms from the same point targets.

For a homogeneously distributed target, all point targets have an identical expected RCS, $\sigma = E[\sigma_i]$. Then, (61) becomes

$$\begin{aligned}
 E[i(\eta)] &= \sum_{i=1}^N \sigma s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \\
 &= \sigma \sum_{i=1}^N s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i). \quad (62)
 \end{aligned}$$

The summation in the final line represents the expected value of the summand under a uniform probability density

$$\begin{aligned}
 E[i(\eta)] &= \sigma \sum_{i=1}^N s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \\
 &= \sigma N E \left[s_{\text{az,focused},r}(\eta'_i) s_{\text{az,focused},s}^*(\eta'_i) \right] \quad (63)
 \end{aligned}$$

where $\eta'_i = \eta - \eta_i$. The function inside the expectation operator represents the InSAR signal of the point target with unit reflectivity located at the origin. The phase term of the expected value, as given by (51), approximately equals the phase at the effective InSAR center of the above point target

$$\arg \{ E [s_{\text{az,focused},r}(\eta'_i) s_{\text{az,focused},s}^*(\eta'_i)] \} \approx \arg \{ i_0(\tilde{\eta}) \} \quad (64)$$

where i_0 and $\tilde{\eta}$ denote the InSAR signal and the effective InSAR center of the point target with unit reflectivity located at the origin, respectively. Therefore, the effective InSAR center of the homogeneous distributed target coincides with the effective InSAR center of the point target located at the origin. The argument $\arg \{ i_0(\tilde{\eta}) \}$ corresponds to the multilooked InSAR phase of the target.

For a heterogeneous target, the RCS of the individual point targets differs from one another. Consequently, the expected value of the interferogram for the target is

$$E[i(\eta)] = E [s_{\text{DS},r}(\eta) s_{\text{DS},s}^*(\eta)]$$

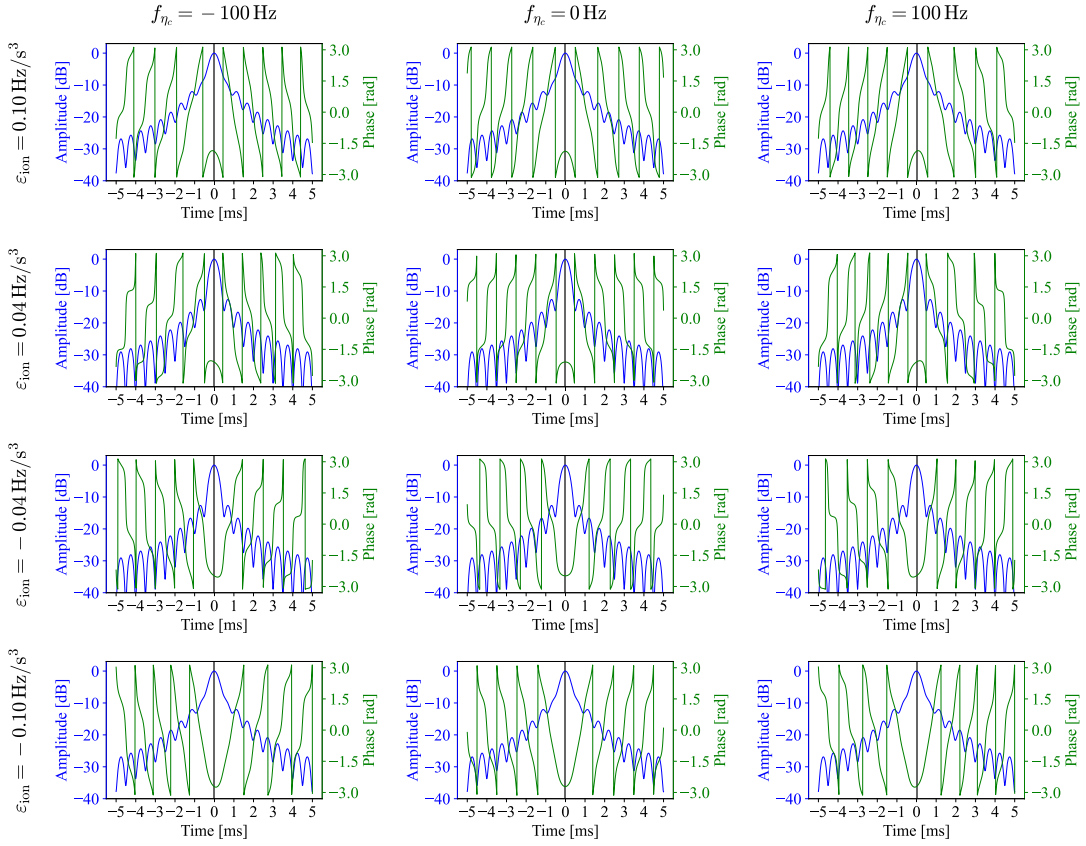


Fig. 14. Focused target with different quartic ionospheric phase components ($\epsilon_{\text{ion}} = \pm 0.04, \pm 0.10 \text{ Hz/s}^3$) under different Doppler centroid frequencies ($f_{\eta_c} = 0, \pm 100 \text{ Hz}$). The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

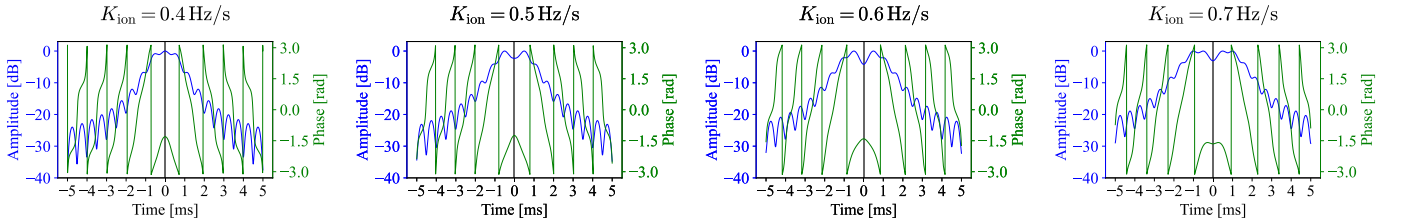


Fig. 15. Focused target with large K_{ion} values. f_{η_c} is 0 Hz. The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the axis of symmetry of the amplitude curve.

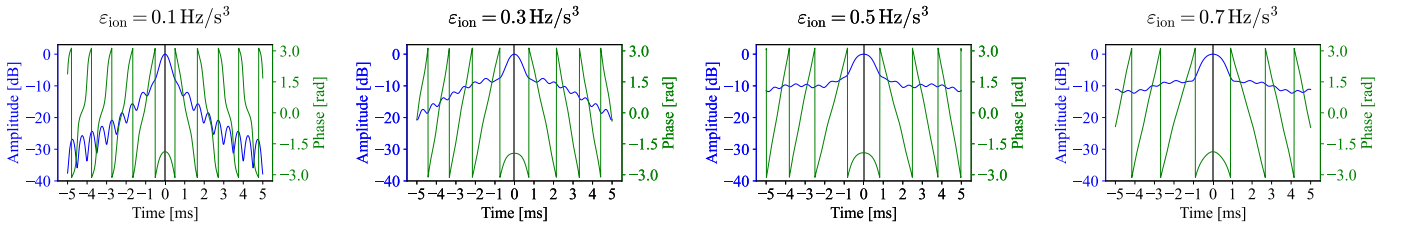


Fig. 16. Focused target with large ϵ_{ion} values. f_{η_c} is 0 Hz. The blue and green curves show the amplitude and phase, respectively, and the black vertical line marks the peak position.

$$\begin{aligned}
 &= \sum_{i=1}^N E[\sigma_i] s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \\
 &= \sum_{i=1}^N \sigma(\eta_i) s_{\text{az,focused},r}(\eta - \eta_i) s_{\text{az,focused},s}^*(\eta - \eta_i) \\
 &= \sum_{i=1}^N \sigma'(\eta'_i) s_{\text{az,focused},r}(\eta'_i) s_{\text{az,focused},s}^*(\eta'_i) \quad (65)
 \end{aligned}$$

where the RCS of the point target located at η_i is $\sigma(\eta_i)$, $\eta'_i = \eta - \eta_i$, and $\sigma'(\eta'_i) = \sigma(\eta - \eta_i)$. The summation in the final step represents the expected value of the summand under a nonuniform probability density that is determined by the RCS distribution. Consequently, its phase differs from that at the effective InSAR center of a point target located at the origin. Nevertheless, a location $\tilde{\eta}'$ can still be found where the phase matches that of the summation result. This location $\tilde{\eta}'$

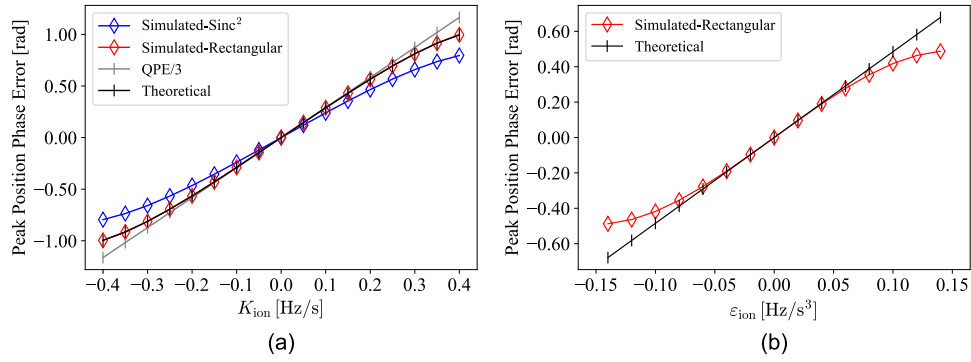


Fig. 17. Phase error at the peak position of the focused target in the presence of (a) quadratic and (b) quartic ionospheric phase components. f_{η_c} is 0 Hz.

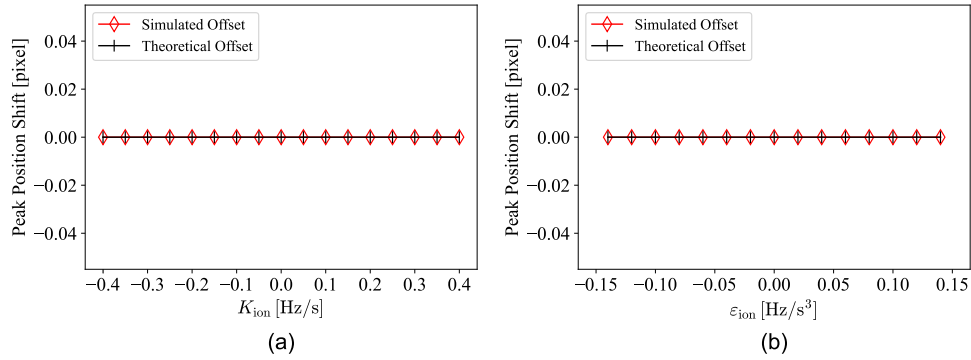


Fig. 18. Shift in the peak position of the focused target in the presence of (a) quadratic and (b) quartic ionospheric phase components. f_{η_c} is 0 Hz.

is defined as the effective InSAR center of the heterogeneous distributed target. The argument $\arg\{i_0(\tilde{\eta}')\}$ corresponds to the multilooked InSAR phase of the target.

In summary, for a homogeneous distributed target, the effective InSAR center coincides with that of the point target located at the origin. In contrast, the effective InSAR center of a heterogeneous distributed target may deviate from this position. The multilooked InSAR phase of the distributed target with effective InSAR center $\tilde{\eta}$ is given by $\arg\{i_0(\tilde{\eta})\}$, where i_0 denotes the InSAR signal of the point target located at the origin. The ionosphere-induced InSAR phase error can then be computed using (54) in Section III-C. Therefore, the distributed and point targets can be unified under the same formulation.

E. Closure Phase Error

To examine the closure phase errors, we consider three SAR signals, denoted by i , j , and k , where i serves as the reference signal, and the other two are coregistered to it using geometric offsets. The phase errors induced by all the even-order ionospheric phase components of the signals are $\Phi_{az,focused,i}^{(even)}(\eta)$, $\Phi_{az,focused,j}^{(even)}(\eta)$, and $\Phi_{az,focused,k}^{(even)}(\eta)$. From these SAR signals, a total of three InSAR pairs can be formed. Their effective InSAR centers are located at $\tilde{\eta}_{i,j}$, $\tilde{\eta}_{i,k}$, and $\tilde{\eta}_{j,k}$, respectively. Further assume that offsets estimated via cross correlation are applied to account for the peak position shifts induced by the odd-order ionospheric phase components, leading to time offsets $\Delta\eta_{i,j}$ and $\Delta\eta_{i,k}$ in the corresponding effective InSAR centers of the secondary signals.

Following (54), the InSAR phase errors at the effective InSAR centers are:

$$\begin{cases} \Phi_{int,i,j}(\tilde{\eta}_{i,j}) = -2\pi f_{\eta_c} \Delta\eta_{i,j} + \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,j}) \\ \quad - \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{i,j} + \Delta\eta_{i,j}) \\ \Phi_{int,i,k}(\tilde{\eta}_{i,k}) = -2\pi f_{\eta_c} \Delta\eta_{i,k} + \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,k}) \\ \quad - \Phi_{az,focused,k}^{(even)}(\tilde{\eta}_{i,k} + \Delta\eta_{i,k}) \\ \Phi_{int,j,k}(\tilde{\eta}_{j,k}) = -2\pi f_{\eta_c} (-\Delta\eta_{i,j} + \Delta\eta_{i,k}) \\ \quad + \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,j}) \\ \quad - \Phi_{az,focused,k}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,k}) \end{cases} \quad (66)$$

The closure phase error in the InSAR phase triplet is then given by

$$\begin{aligned} & \Phi_{closure,i,j,k}(\tilde{\eta}_{i,j,k}) \\ &= \Phi_{int,i,j}(\tilde{\eta}_{i,j}) + \Phi_{int,j,k}(\tilde{\eta}_{j,k}) - \Phi_{int,i,k}(\tilde{\eta}_{i,k}) \\ &= -2\pi f_{\eta_c} (\Delta\eta_{i,j} - \Delta\eta_{i,j} + \Delta\eta_{i,k} - \Delta\eta_{i,k}) \\ & \quad + \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,j}) - \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{i,j} + \Delta\eta_{i,j}) \\ & \quad + \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,j}) \\ & \quad - \Phi_{az,focused,k}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,k}) \\ & \quad - \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,k}) \\ & \quad + \Phi_{az,focused,k}^{(even)}(\tilde{\eta}_{i,k} + \Delta\eta_{i,k}) \\ &= \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,j}) - \Phi_{az,focused,i}^{(even)}(\tilde{\eta}_{i,k}) \\ & \quad - \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{i,j} + \Delta\eta_{i,j}) \\ & \quad + \Phi_{az,focused,j}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,j}) \\ & \quad - \Phi_{az,focused,k}^{(even)}(\tilde{\eta}_{j,k} + \Delta\eta_{i,k}) \end{aligned}$$

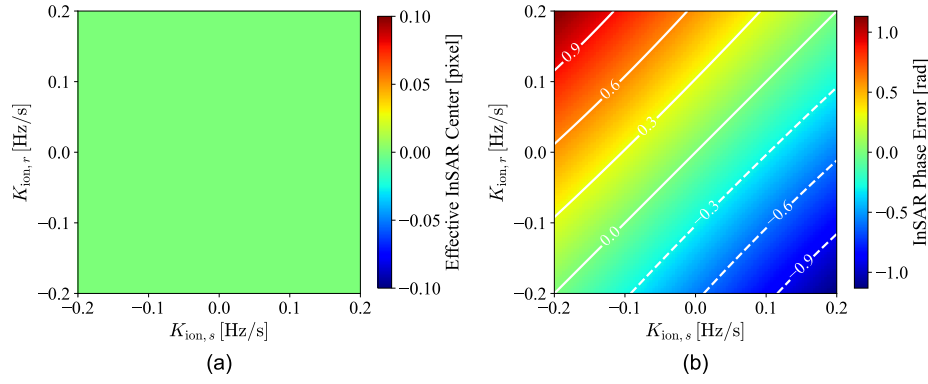


Fig. 19. Simulated InSAR result in the presence of quadratic ionospheric phase components. Quadratic ionospheric phase components ($K_{\text{ion},r}$ and $K_{\text{ion},s}$) are introduced into the reference and secondary signals. (a) Effective InSAR center. (b) InSAR phase error. f_{η_c} is 0 Hz and PRF is 2400 Hz.

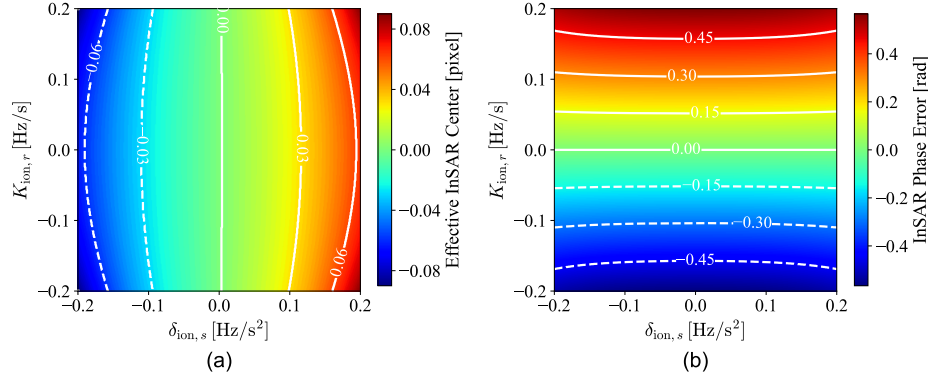


Fig. 20. Simulated InSAR result in the presence of quadratic and cubic ionospheric phase components. Quadratic ionospheric phase components ($K_{\text{ion},r}$) are introduced into the reference signal, whereas cubic ionospheric phase components ($\delta_{\text{ion},s}$) are introduced into the secondary signal. (a) Effective InSAR center. (b) InSAR phase error. f_{η_c} is 0 Hz and PRF is 2400 Hz.

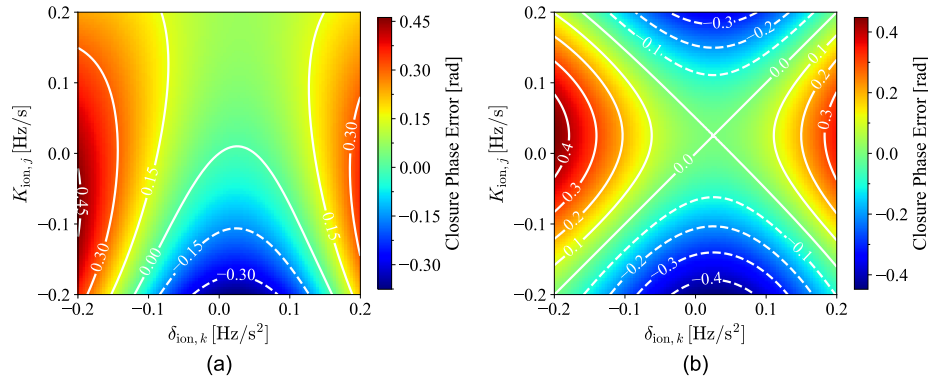


Fig. 21. Simulated closure phase errors from three SAR signals s_i , s_j , and s_k . A quadratic ionospheric phase component with a fixed coefficient ($K_{\text{ion},i}$) of 0.4 Hz/s is introduced into s_i . (a) Quadratic ionospheric phase components ($K_{\text{ion},j}$) are introduced into s_j , whereas cubic ionospheric phase components ($\delta_{\text{ion},k}$) are introduced into s_k . (b) Cubic ionospheric phase components ($\delta_{\text{ion},j}$ and $\delta_{\text{ion},k}$) are introduced into both s_j and s_k . f_{η_c} is 0 Hz.

$$+ \Phi_{\text{az,focused},k}^{(\text{even})}(\tilde{\eta}_{i,k} + \Delta\eta_{i,k}). \quad (67)$$

From the above expression, the coupling between the InSAR phase errors and the variations in the effective InSAR center can induce a closure phase error. The InSAR phase errors are mainly induced by the even-order ionospheric phase components, whereas the variations of the effective InSAR center are mainly induced by the odd-order ionospheric phase components. The defocusing induced by the 2nd-order and higher order ionospheric phase components also contributes

to the variations. The linear phase induced by the Doppler centroid frequency, however, does not affect the closure phase error. It is noteworthy that if the signals are well coregistered, the effective InSAR center does not vary, and

$$\tilde{\eta}_{i,j} = \tilde{\eta}_{j,k} = \tilde{\eta}_{i,k}. \quad (68)$$

Then, the closure phase error becomes zero. This highlights the importance of precise azimuth coregistration. To mitigate the closure phase error, spectral-diversity [30] or

TABLE I
DETAILS OF THE EXPERIMENTAL DATA

Satellite	Mode	Region	Ionosphere	Date [YYYY/MM/DD]		
ALOS	FBS/FBD	Chile	Heavy	2007/02/26	2007/08/29	2007/10/14
ALOS-2	UBS	Mount Fuji, Japan	Moderate	2020/06/19	2021/06/18	2021/07/16
		Kumamoto, Japan	Light	2018/11/12	2018/12/10	2019/02/18

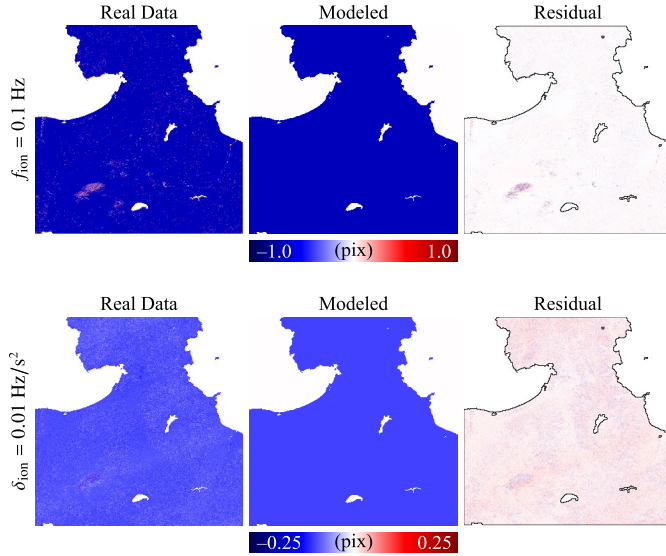


Fig. 22. Azimuth offsets induced by a linear ionospheric phase component with $f_{\text{ion}} = 0.1$ Hz and a cubic ionospheric phase component with $\delta_{\text{ion}} = 0.01$ Hz/s².

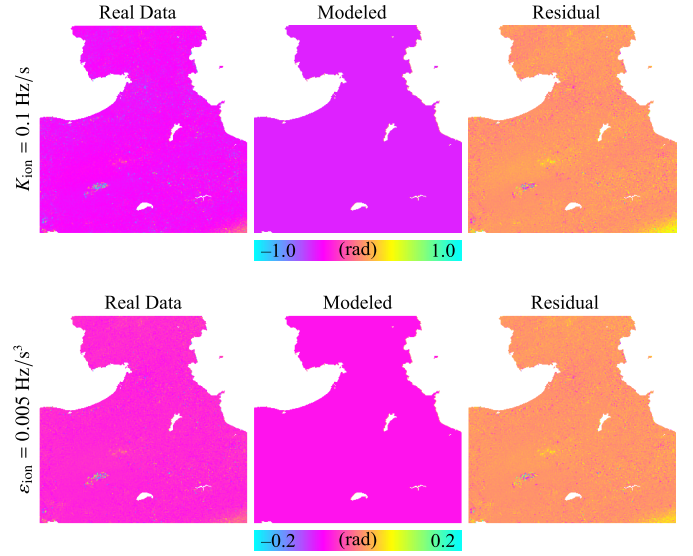


Fig. 23. InSAR phase errors induced by a quadratic ionospheric phase component with $K_{\text{ion}} = 0.1$ Hz/s and a quartic ionospheric phase component with $\epsilon_{\text{ion}} = 0.005$ Hz/s³.

cross correlation methods can be applied following geometric coregistration.

We again consider only the quadratic ionospheric phase component to illustrate the closure phase error. Fig. 7 presents the results under typical system configurations. The secondary signals have been coregistered to the reference signal using geometric offsets.

IV. SIMULATION

In this section, we conduct simulations to verify the effects of the along-track ionospheric variations within the synthetic aperture, as derived in Sections II and III. We use the parameters of an L-band satellite SAR system. The azimuth FM rate K_a is 600 Hz/s, the azimuth bandwidth B is 2000 Hz, and the effective radar velocity is 7557.8 m/s. The simulation begins with the generation of the raw azimuth signal using (1). Different ionospheric phase components are introduced into the signal. The resulting signals are then focused by a matched filter constructed using the nominal FM rate. The results are presented below.

A. Effects of Odd-Order Ionospheric Phase Components

Fig. 8 presents examples of the focused target with different linear ionospheric phase components (f_{ion}) under different

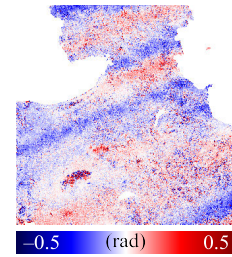


Fig. 24. Closure phase errors induced by a quadratic ionospheric phase component with $K_{\text{ion}} = 0.2$ Hz/s.

Doppler centroid frequencies (f_{η_c}). The results for cubic ionospheric phase components (δ_{ion}) are shown in Fig. 9. The azimuth envelope is modeled as a rectangular function. As expected, no phase errors are introduced into the focusing results, but the peak positions are shifted, which depends on the ionospheric phase component coefficients. The slope of the phase varies as a function of the Doppler centroid frequency f_{η_c} . For the linear ionospheric phase components, the resulting amplitude profiles are not altered. However, they are distorted by the cubic ionospheric phase components, leading to defocusing. The defocus is barely visible in Fig. 9. As δ_{ion} increases, however, the defocus becomes pronounced, as shown in Fig. 10. The main lobe is broadened and merges with the sidelobes on one side.

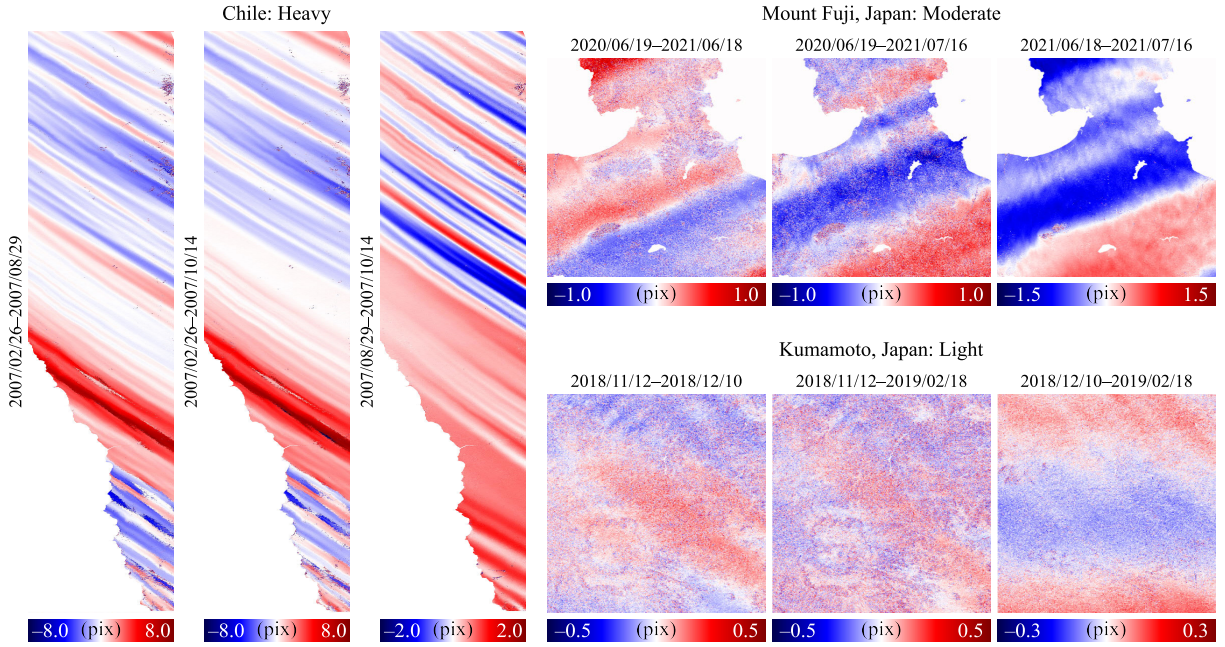


Fig. 25. Azimuth misregistrations induced by the odd-order ionospheric phase components.

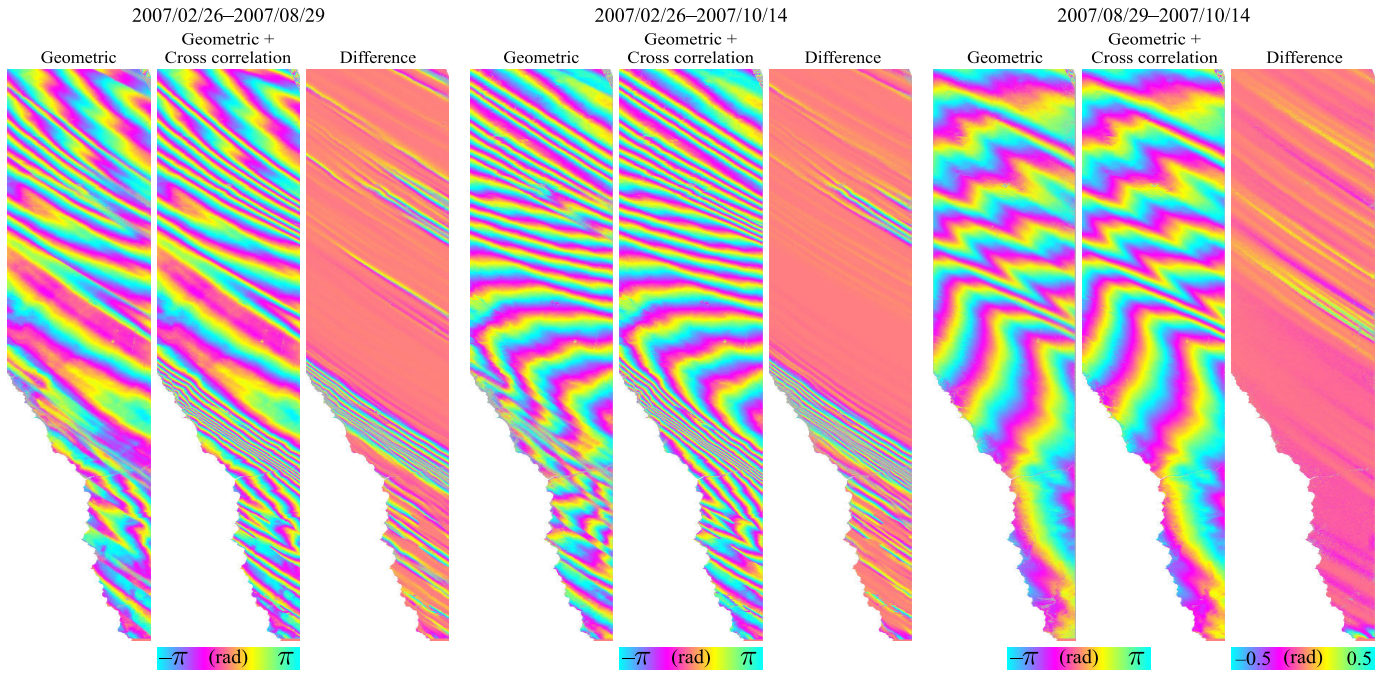


Fig. 26. Impacts of the azimuth misregistrations induced by the odd-order ionospheric phase components on InSAR phase errors. Region: Chile.

The shifts of the peak positions obtained from simulations are compared with theoretical predictions from (14) and (46) in Fig. 11, where f_{η_c} is fixed at 0 Hz. For the linear ionospheric phase components, they exhibit excellent agreement. For the cubic ionospheric phase components, (46) provides a good approximation for $|\delta_{\text{ion}}| < 0.4 \text{ Hz/s}^2$, but it begins to overestimate the shift as δ_{ion} increases. As discussed in Section II-B, 1 Hz is already a large value for f_{ion} . We expect that δ_{ion} should be much smaller than f_{ion} . Therefore, $|\delta_{\text{ion}}| > 0.4 \text{ Hz/s}^2$ rarely occurs. The phases at the peak positions obtained from simulations are also in excellent agreement with the theoretical

predictions from (16) and (37) in Fig. 12, where f_{η_c} is fixed at 100 Hz.

B. Effects of Even-Order Ionospheric Phase Components

Fig. 13 presents examples of the focused target with different quadratic ionospheric phase components (K_{ion}) under different Doppler centroid frequencies (f_{η_c}). The results for quartic ionospheric phase components (ε_{ion}) are shown in Fig. 14. The azimuth envelope is approximated by a rectangular function. The results confirm that even-order ionospheric

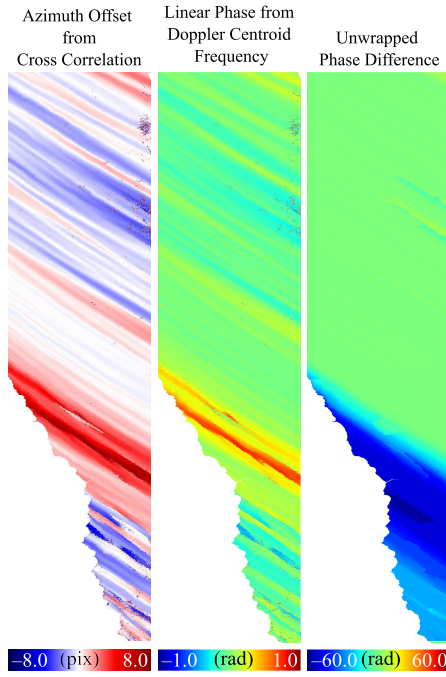


Fig. 27. Azimuth offset from cross correlation, the phase difference caused by the linear phase associated with the Doppler centroid frequency (33 Hz), and the unwrapped phase difference for the InSAR pair 2007/02/26–2007/08/29. Region: Chile.

phase components do not shift the peak position of the focused target. However, they induce phase anomalies whose magnitudes depend on the ionospheric phase component coefficients K_{ion} and ε_{ion} . The Doppler centroid frequency f_{η_c} breaks the symmetry of the phase profile. The amplitude profile is distorted with increasing ionospheric phase component coefficients K_{ion} and ε_{ion} , manifesting as main lobe broadening and sidelobe merging. In addition, as illustrated in Figs. 15 and 16, the main lobe in the presence of a quadratic ionospheric phase component splits into two distinct peaks when K_{ion} exceeds a threshold, which is a phenomenon not observed with the quartic ionospheric phase component. The distortion of the amplitude profile leads to defocusing of the target.

The phase error at the peak position of the focused target is subsequently examined in Fig. 17 with f_{η_c} fixed at 0 Hz. Fig. 17(a) presents the results for the quadratic ionospheric phase component, comparing the phase error from simulation, using both rectangular and $\text{sinc}^2(\cdot)$ azimuth envelopes, against that predicted by (22) derived in Section II-C and the QPE/3 from [26]. The results demonstrate excellent agreement between the simulation for a rectangular envelope and the theory. However, the phase errors from the simulation using the $\text{sinc}^2(\cdot)$ envelope are slightly lower than the theoretical predictions, and the QPE/3 values are slightly higher. This discrepancy widens as K_{ion} increases. Fig. 17(b) shows the results for the quartic ionospheric phase component. Equation (47) provides a good approximation for $|\varepsilon_{\text{ion}}| < 0.1 \text{ Hz/s}^3$, but it increasingly overestimates the phase error as ε_{ion} increases.

The peak positions of the focused targets are also examined in Fig. 18. As predicted in Sections II-C and II-D, no shift is observed in the simulations.

C. InSAR Phase Error

Figs. 19 and 20 present the simulated results of the effective InSAR centers and the InSAR phase errors in the presence of different ionospheric phase components in the reference and secondary signals. We have used the peak position of the interferogram to represent the effective InSAR center. The azimuth envelopes are simplified as rectangular functions. The Doppler centroid frequency f_{η_c} is fixed at 0 Hz.

In Fig. 19, quadratic ionospheric phase components ($K_{\text{ion},r}$ and $K_{\text{ion},s}$) are introduced into the reference and secondary signals. Fig. 19(a) shows that the effective InSAR centers remain unchanged, which agrees with the theoretical prediction that the even-order ionospheric phase components do not shift the peak position. In contrast, as shown in Fig. 19(b), the InSAR phase errors vary significantly with the quadratic ionospheric phase components.

In Fig. 20, quadratic ionospheric phase components ($K_{\text{ion},r}$) are introduced into the reference signal, while cubic ionospheric phase components ($\delta_{\text{ion},s}$) are introduced into the secondary signal. As shown in Fig. 20(a), the effective InSAR center varies due to the cubic ionospheric phase components, which agrees with the theoretical prediction that the odd-order ionospheric phase components shift the peak position. Defocusing also exerts a subtle influence on the effective InSAR center. As expected, the InSAR phase error also varies significantly with the ionospheric phase components, as shown in Fig. 20(b).

D. Closure Phase Error

Fig. 21 presents the simulated results of the closure phase errors in the presence of different ionospheric phase components. The closure phase errors are calculated from three signals denoted by s_i , s_j , and s_k . The azimuth envelopes are simplified as a rectangular function. Similar to the simulations in Section IV-C, we take the quadratic and cubic ionospheric phase components as examples. A quadratic ionospheric phase component with a fixed coefficient ($K_{\text{ion},i}$) of 0.4 Hz/s is introduced into s_i . The ionospheric phase components in s_j and s_k are varied independently. The Doppler centroid frequency f_{η_c} is fixed at 0 Hz.

In Fig. 21(a), quadratic ionospheric phase components ($K_{\text{ion},j}$) are introduced into s_j , while cubic ionospheric phase components ($\delta_{\text{ion},k}$) are introduced into s_k . In Fig. 21(b), cubic ionospheric phase components ($\delta_{\text{ion},j}$ and $\delta_{\text{ion},k}$) are introduced into both s_j and s_k . As expected, both results show closure phase errors that are not negligible.

E. Simulation With Real SAR Data

We further quantitatively validate the theoretical findings by introducing additional ionospheric phase components into the secondary SAR image of InSAR pairs in the range–Doppler domain. We use the data acquired by ALOS-2 over the Mount Fuji area in Japan. The details of the data are listed in Table I. We first examine the azimuth misregistrations and InSAR phase errors using the InSAR pair of 2020/06/19–2021/06/18.

Fig. 22 presents the azimuth offsets induced by a linear ionospheric phase component with a coefficient (f_{ion}) of 0.1 Hz

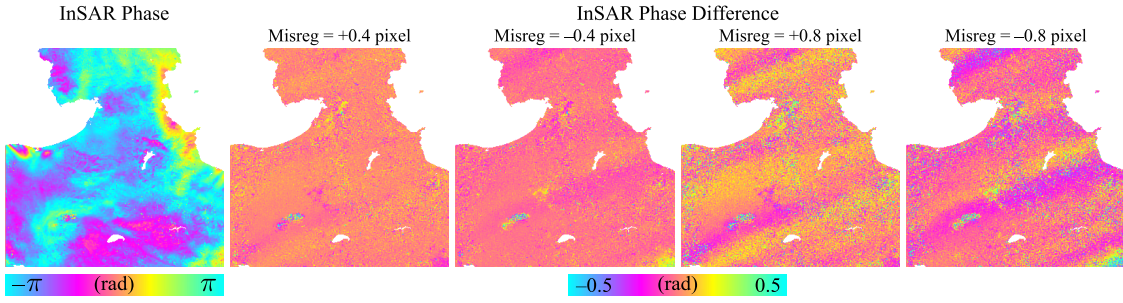


Fig. 28. InSAR phases of the Mount Fuji region. The InSAR pair used is 2020/06/19–2021/06/18. The first image shows the original InSAR phase generated using combined geometric and cross correlation offsets. The other images show the differences between the InSAR phases generated by intentionally adding spatially uniform azimuth misregistrations and the original InSAR phase.

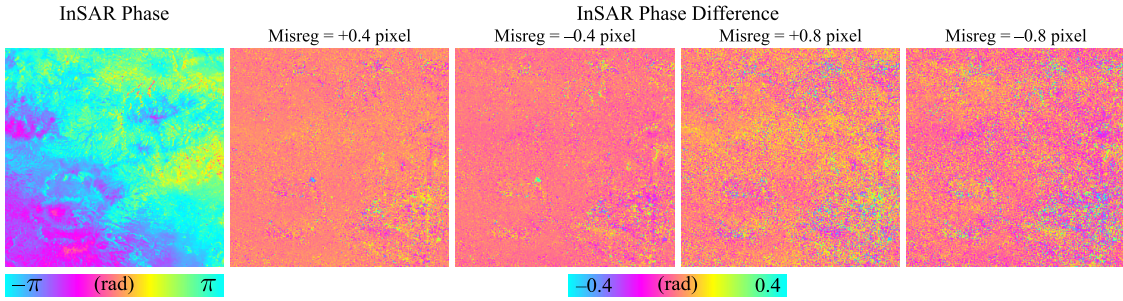


Fig. 29. InSAR phases of Kumamoto region. The InSAR pair used is 2018/11/12–2018/12/10. The first image shows the original InSAR phase generated using combined geometric and cross correlation offsets. The other images show the differences between the InSAR phases generated by intentionally adding spatially uniform azimuth misregistrations and the original InSAR phase.

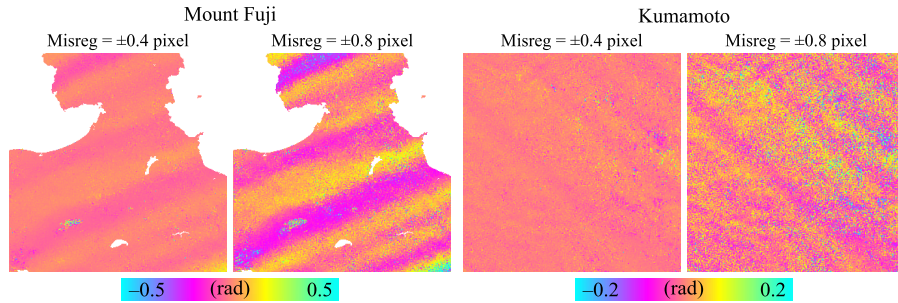


Fig. 30. Sums of the phase differences.

and a cubic ionospheric phase component with a coefficient (δ_{ion}) of 0.01 Hz/s^2 . The observed values are consistent with the theoretical predictions.

Fig. 23 shows the resulting InSAR phase errors after introducing a quadratic ionospheric phase component with a coefficient (K_{ion}) of 0.1 Hz/s and a quartic ionospheric phase component with a coefficient (ε_{ion}) of 0.005 Hz/s^3 . These InSAR phase errors are also in good agreement with the theoretical predictions.

Finally, we examine the closure phase errors using the Mount Fuji dataset listed in Table I. A quadratic ionospheric phase component with a coefficient (K_{ion}) of 0.2 Hz/s is introduced into the range–Doppler domain of the 2021/06/18 acquisition. This introduction, combined with the preexisting ionospheric effects in the data, shifts the effective InSAR centers of the pairs 2020/06/19–2021/06/18 and 2021/06/18–2021/07/16, leading to closure phase errors. The resulting closure phase errors are presented in Fig. 24. How-

ever, accurately predicting these errors remains challenging because the preexisting ionospheric effects are unknown, which consequently renders the shifts in the effective InSAR centers undetermined. Nevertheless, the results exhibit ionospheric “streaks,” which align with the azimuth misregistrations shown in Fig. 25.

V. REAL-DATA EXPERIMENT

For real-data experiments, we use L-band ALOS PALSAR and ALOS-2 PALSAR-2 data, although the developed theory is also applicable to SAR data acquired by other satellites. The details of these datasets are summarized in Table I.

A. Azimuth Misregistration

To examine the azimuth misregistrations induced by the odd-order ionospheric phase components, we coregister the

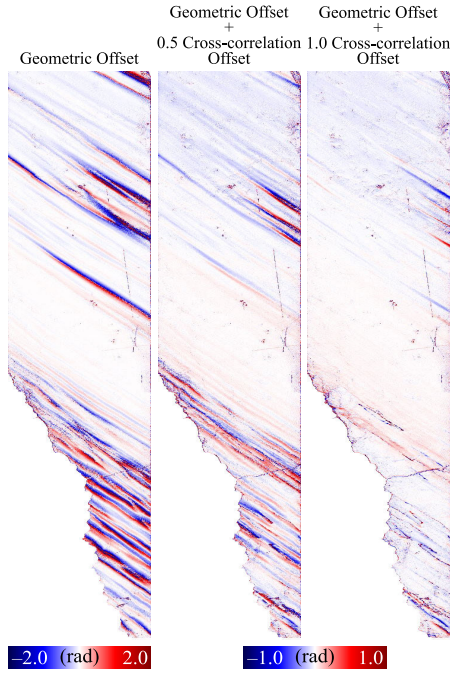


Fig. 31. Closure phases of Chile region.

SAR images using geometric offsets and then conduct cross correlation between the coregistered images.

Fig. 25 presents the azimuth offsets from cross correlation, representing the azimuth misregistrations induced by the ionosphere. The azimuth offsets directly reflect the effects of the odd-order ionospheric phase components. They dominate the variation of the effective InSAR center and, consequently, exert a significant impact on both InSAR and closure phase errors. However, it remains challenging to consistently estimate these spatially varying azimuth offsets across the entire image with high precision in practical scenarios.

B. InSAR Phase Error

We first examine the impacts of the azimuth misregistrations induced by the odd-order ionospheric phase components on the InSAR phase errors. Fig. 26 presents the results for Chile dataset, in which acquisition 2007/02/26 is subject to heavy ionospheric effects. We generate interferograms by coregistering the images using two strategies: 1) geometric offsets and 2) geometric and cross correlation offsets. The differences between the interferograms are caused by the combined effects of the azimuth misregistrations induced by the odd-order ionospheric phase components, the phase errors induced by the even-order ionospheric phase components, and the linear phase induced by the Doppler centroid frequency. Correcting the azimuth misregistrations induced by the odd-order ionospheric phase components leads to variations in the effective InSAR center and, consequently, significantly changes the InSAR phase errors.

Notably, large spatial variations are observed in the phase differences derived from the two coregistration methods for the InSAR pairs 2007/02/26–2007/08/29 and

2007/02/26–2007/10/14. They are unlikely to be caused by the phase errors induced by the even-order ionospheric phase components, as these phase errors are small. We then examine the potential effect of the linear phase induced by the Doppler centroid frequency. The phase difference caused by the linear phase depends on the azimuth offsets induced by the odd-order ionospheric phase components. Fig. 27 shows the azimuth offset from cross correlation, the phase difference caused by the linear phase, and the unwrapped phase difference for pair 2007/02/26–2007/08/29. Since the phase difference caused by the linear phase is considerably smaller than the unwrapped phase difference, the large phase differences are not caused by the linear phase associated with the Doppler centroid frequency. The interferograms formed with such large azimuth misregistrations would completely decorrelate according to classical InSAR theory. However, the fringes remain clearly visible. The underlying mechanism is beyond the scope of this work. Nevertheless, such large azimuth misregistrations are unlikely to occur in practical scenarios, as they would be mitigated by cross correlation.

We now validate the impacts of the even-order ionospheric phase components on InSAR phase errors. We intentionally add spatially uniform azimuth misregistrations of $\pm\Delta\eta$ to the combined geometric and cross correlation offsets. We compute the phase differences between the interferograms formed with and without the intentionally added azimuth misregistrations. The phase differences arise from the phase errors induced by the even-order ionospheric phase components and the linear phase induced by the Doppler centroid frequency. We then compute the sum of the phase differences, which contains only the phase errors induced by the even-order ionospheric phase components. The entire process is described in detail below.

Assuming that the effective InSAR center of a ground target in the reference signal is located at $\tilde{\eta}$. Recall that we have defined the origin of the azimuth time as the zero Doppler time throughout this article. The zero Doppler times of the reference and secondary signals coincide after coregistration using geometric offsets. Here, the offset estimated via cross correlation is also used in coregistration, leading to an azimuth offset $\Delta\eta_r$ in the corresponding effective InSAR center of the secondary signal. Therefore, the effective InSAR center in the secondary signal becomes $\tilde{\eta} + \Delta\eta_r$. According to (52) and (53), the phase errors in the reference and secondary signals at the effective InSAR centers are

$$\begin{cases} \Phi_{\text{az,focused},r}(\tilde{\eta}) = 2\pi f_{\eta_c} \tilde{\eta} + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) \\ \Phi_{\text{az,focused},s}(\tilde{\eta} + \Delta\eta_r) = 2\pi f_{\eta_c} (\tilde{\eta} + \Delta\eta_r) \\ \quad + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{cases} \quad (69)$$

The original InSAR phase error is

$$\begin{aligned} \Phi_{\text{int}}(\tilde{\eta}) &= \Phi_{\text{az,focused},r}(\tilde{\eta}) - \Phi_{\text{az,focused},s}(\tilde{\eta} + \Delta\eta_r) \\ &= -2\pi f_{\eta_c} \Delta\eta_r + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{aligned} \quad (70)$$

If we intentionally add a misregistration $\Delta\eta$ to the offset, the effective InSAR centers shift to $\tilde{\eta}_+$ and $\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta$ in

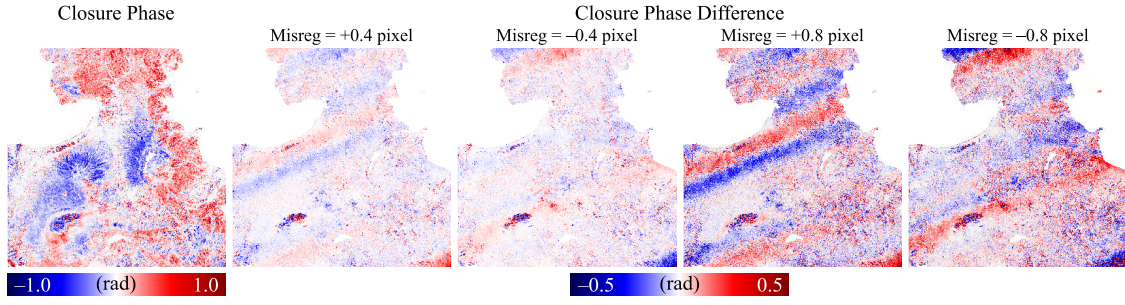


Fig. 32. Closure phases of the Mount Fuji region. The first image shows the original closure phase generated using combined geometric and cross correlation offsets. The other images show the differences between the closure phases generated by intentionally adding spatially uniform azimuth misregistrations and the original closure phase.

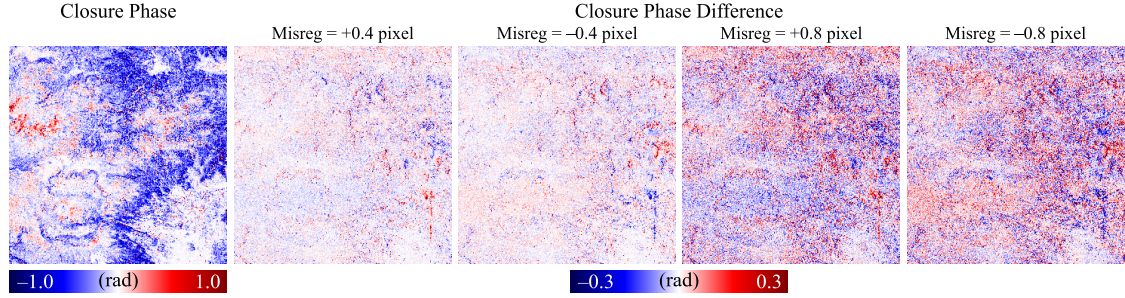


Fig. 33. Closure phases of the Kumamoto region. The first image shows the original closure phase generated using combined geometric and cross correlation offsets. The other images show the differences between the closure phases generated by intentionally adding spatially uniform azimuth misregistrations and the original closure phase.

the reference and secondary signals, and the phase errors at the effective InSAR centers become

$$\begin{cases} \Phi_{\text{az,focused},r,+}(\tilde{\eta}_+) = 2\pi f_{\eta_c} \tilde{\eta}_+ + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_+) \\ \Phi_{\text{az,focused},s,+}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta) \\ = 2\pi f_{\eta_c}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta) \\ + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta). \end{cases} \quad (71)$$

Accordingly, the InSAR phase error is

$$\begin{aligned} \Phi_{\text{int},+}(\tilde{\eta}_+) &= \Phi_{\text{az,focused},r,+}(\tilde{\eta}_+) \\ &\quad - \Phi_{\text{az,focused},s,+}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta) \\ &= -2\pi f_{\eta_c}(\Delta\eta_r + \Delta\eta) + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_+) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta). \end{aligned} \quad (72)$$

The difference between this InSAR phase error and the original InSAR phase error is

$$\begin{aligned} \Delta\Phi_{\text{int},+} &= \Phi_{\text{int},+}(\tilde{\eta}_+) - \Phi_{\text{int}}(\tilde{\eta}) \\ &= -2\pi f_{\eta_c} \Delta\eta + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_+) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta) \\ &\quad - \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{aligned} \quad (73)$$

Similarly, if the added misregistration is $-\Delta\eta$, the effective InSAR centers shift to $\tilde{\eta}_-$ and $\tilde{\eta}_- + \Delta\eta_r - \Delta\eta$ in the reference and secondary signals, and the phase errors at the effective InSAR centers become

$$\begin{cases} \Phi_{\text{az,focused},r,-}(\tilde{\eta}_-) = 2\pi f_{\eta_c} \tilde{\eta}_- + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_-) \\ \Phi_{\text{az,focused},s,-}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta) \\ = 2\pi f_{\eta_c}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta) \\ + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta). \end{cases} \quad (74)$$

Accordingly, the InSAR phase error is

$$\begin{aligned} \Phi_{\text{int},-}(\tilde{\eta}_-) &= \Phi_{\text{az,focused},r,-}(\tilde{\eta}_-) \\ &\quad - \Phi_{\text{az,focused},s,-}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta) \\ &= -2\pi f_{\eta_c}(\Delta\eta_r - \Delta\eta) + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_-) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta). \end{aligned} \quad (75)$$

The difference between this InSAR phase error and the original InSAR phase error is

$$\begin{aligned} \Delta\Phi_{\text{int},-} &= \Phi_{\text{int},-}(\tilde{\eta}_-) - \Phi_{\text{int}}(\tilde{\eta}) \\ &= 2\pi f_{\eta_c} \Delta\eta + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_-) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta) \\ &\quad - \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) + \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{aligned} \quad (76)$$

The sum of the two phase differences is

$$\begin{aligned} \Delta\Phi_{\text{int},\pm} &= \Delta\Phi_{\text{int},+} + \Delta\Phi_{\text{int},-} \\ &= \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_+) + \Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}_-) \\ &\quad - 2\Phi_{\text{az,focused},r}^{(\text{even})}(\tilde{\eta}) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_+ + \Delta\eta_r + \Delta\eta) \\ &\quad - \Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta}_- + \Delta\eta_r - \Delta\eta) \\ &\quad + 2\Phi_{\text{az,focused},s}^{(\text{even})}(\tilde{\eta} + \Delta\eta_r). \end{aligned} \quad (77)$$

In the sum of the phase differences, the linear phases induced by the Doppler centroid frequency cancel each other. In contrast, the phase errors induced by the even-order ionospheric phase components do not cancel and thus remain.

We conduct experiments using the Mount Fuji and Kumamoto datasets. The original interferograms are generated

by coregistering the SAR images using geometric and cross correlation offsets. We test $\pm\Delta\eta = \pm 0.4$ pixel and $\pm\Delta\eta = \pm 0.8$ pixel. We have removed the constant ionospheric phase component estimated using the range split-spectrum method. The phase differences for the two datasets are presented in Figs. 28 and 29. The sums of the phase differences shown in Fig. 30 exhibit more pronounced variations compared with the individual phase differences, indicating the impact of the even-order ionospheric phase components.

C. Closure Phase Error

Fig. 31 presents the closure phases for Chile dataset. The closure phases are derived using three sets of interferograms, formed using the geometric offsets with: 1) no additional offset; 2) a half cross correlation offset; and 3) a full cross correlation offset, respectively. The closure phases manifest as “streaks,” a distinct signature of ionospheric effects. They are also correlated with the azimuth misregistrations induced by the odd-order ionospheric phase components, as shown in Fig. 25. As the azimuth misregistrations are progressively mitigated through cross correlation, the resulting closure phases are correspondingly reduced. This validates the theory in Section III-E that precise azimuth coregistration can eliminate closure phase errors related to the along-track ionospheric variations within the synthetic aperture.

Fig. 32 presents the closure phases for the Mount Fuji dataset. We have coregistered the images using the combined geometric and cross correlation offsets. In addition, the ionospheric effects in this dataset are moderate. Therefore, the closure phase errors attributable to the ionosphere are not significant and are overwhelmed by the closure phases of physical origin on the ground. Nevertheless, a banding feature spatially correlated with the azimuth misregistrations in Fig. 25 remains discernible. It is likely attributable to the ionosphere, arising from the residual azimuth misregistrations that persist even after the application of cross correlation offsets. As in Section V-B, we intentionally add spatially uniform azimuth misregistrations to the acquisition 2021/06/18 and calculate the resulting closure phases. The closure phase differences relative to the original results exhibit distinct ionospheric signatures.

Fig. 33 presents the corresponding results for Kumamoto dataset, which is subject to light ionospheric effects. The spatially uniform azimuth misregistrations are added to the acquisition 2018/12/10. The results are similar to those of the Mount Fuji dataset, though the ionospheric effects are even less pronounced.

The results in this section collectively demonstrate the significant impacts of the along-track ionospheric variations within the synthetic aperture on both InSAR and closure phases.

VI. CONCLUSION

In this work, we have systematically investigated the impacts of the along-track ionospheric variations within the synthetic aperture. For the focusing results of a point target, the individual effects of different ionospheric phase components are as follows.

- 1) The odd-order ionospheric phase components (e.g., 1st, 3rd, 5th,...) induce a shift in the peak position of the focused target, but do not change its phase.
- 2) The even-order ionospheric phase components (e.g., 2nd, 4th, 6th,...) induce an error in its phase, but do not shift its peak position. The phase error is an even function that is symmetric about the peak position. In the absence of the peak position shift induced by odd-order ionospheric phase components, the phase error is symmetric about zero Doppler position ($\eta = 0$).
- 3) The 2nd-order and higher order ionospheric phase components also result in defocusing of the target, a well-recognized phenomenon.

Together with the linear phase induced by the Doppler centroid frequency, which is an odd function that is symmetric about the zero Doppler position ($\eta = 0$), these impacts can lead to significant InSAR and closure phase errors depending on the ionospheric activity level. Precise azimuth coregistration can effectively mitigate the closure phase errors. We reiterate that the phase error is computed between the signals at identical azimuth positions with respect to the same origin, even if the peak position is shifted. The coupling effects among different components, however, are complex and should be further investigated in future work.

While it is difficult to establish a numerical threshold or objective criterion to determine when the analyzed ionospheric effects become negligible, they are generally prevalent and nonnegligible at L-band as evidenced by the frequently observed azimuth offsets comparable to the azimuth pixel size. For C-band SAR, the corresponding ionospheric effects are typically negligible.

APPENDIX A RANGE-AZIMUTH COUPLING

In this section, we examine the range-azimuth coupling issues associated with ionospheric effects.

The first issue concerns the imaging geometry. The imaging geometries are fundamentally distinct in the range and azimuth directions. For a point target, the piercing point of the SAR signal in the ionosphere is fixed in the range direction. The resulting ionospheric effects therefore do not vary for the target. In the azimuth direction, however, the piercing point moves continuously within the synthetic aperture, leading to spatially varying ionospheric effects. Consequently, there is no range-azimuth coupling in this regard.

The second issue is related to azimuth SAR focusing, which involves range cell migration (RCM) correction (RCMC) and secondary range compression (SRC). As RCMC and SRC depend on azimuth phase history, they may be affected by ionospheric effects, resulting in range-azimuth coupling effects. However, we demonstrate that these effects are negligible. In the following, we will take the linear ionospheric phase component, which is typically the largest contribution from the ionosphere, as an example and provide a numerical assessment.

We begin with RCMC. Assume that the SAR signal experiences an azimuth-varying ionospheric TEC profile, $\text{TEC}(\eta)$, which induces a linear ionospheric phase component

with a rate of f_{ion} . The resulting signal can be expressed as [26]

$$\begin{aligned} s_{\text{ion}}(\tau, \eta) &= A_0 w_r \left[\tau - \frac{2R(\eta)}{c} \right] w_a(\eta - \eta_c) \exp \left\{ -j \frac{4\pi f_0 R(\eta)}{c} \right\} \\ &\quad \times \exp \left\{ j\pi K_r \left[\tau - \frac{2R(\eta)}{c} \right]^2 \right\} \exp \left\{ j \frac{4\pi K_{\text{TEC}}(\eta)}{c f_0} \right\} \\ &= A_0 w_r \left[\tau - \frac{2R(\eta)}{c} \right] w_a(\eta - \eta_c) \exp \left\{ -j \frac{4\pi f_0 R(\eta)}{c} \right\} \\ &\quad \times \exp \left\{ j\pi K_r \left[\tau - \frac{2R(\eta)}{c} \right]^2 \right\} \exp \{ j2\pi f_{\text{ion}}(\eta - \eta_c) \} \quad (78) \end{aligned}$$

where w_r is the range envelope, τ is the range time, f_0 is the radar central frequency, $R(\eta)$ is the instantaneous slant range, and K_r is the range chirp FM rate. Other parameters are identical to those in (1). After range compression, the signal in the range–Doppler domain is given by

$$\begin{aligned} S_{\text{ion}}(\tau, f_\eta) &= A_0 p_r \left[\tau - \frac{2R_{\text{rd}}(f_\eta - f_{\text{ion}})}{c} \right] W_a(f_\eta - f_{\eta_c} - f_{\text{ion}}) \\ &\quad \times \exp \left\{ -j \frac{4\pi f_0 R_0}{c} \right\} \exp \left\{ j\pi \frac{(f_\eta - f_{\text{ion}})^2}{K_a} \right\} \\ &\quad \times \exp \left\{ j2\pi \frac{f_{\eta_c} f_{\text{ion}}}{K_a} \right\} \quad (79) \end{aligned}$$

where

$$\begin{aligned} R_{\text{rd}}(f_\eta - f_{\text{ion}}) &\approx R_0 + \frac{\lambda^2 R_0}{8V_r^2} (f_\eta - f_{\text{ion}})^2 \\ &= R_0 + \Delta R(f_\eta - f_{\text{ion}}), \quad (80) \end{aligned}$$

p_r is the compressed range envelope, and f_η is the azimuth frequency. Other parameters are identical to those in (1)–(3). The RCM component to be corrected, $\Delta R(f_\eta - f_{\text{ion}})$, is indeed influenced by the ionosphere. The resulting RCMC error is

$$\begin{aligned} \Delta R(f_\eta - f_{\text{ion}}) - \Delta R(f_\eta) &= \frac{\lambda^2 R_0}{8V_r^2} (f_\eta - f_{\text{ion}})^2 - \frac{\lambda^2 R_0}{8V_r^2} f_\eta^2 \\ &= \frac{\lambda^2 R_0}{8V_r^2} (f_{\text{ion}}^2 - 2f_{\text{ion}}f_\eta). \quad (81) \end{aligned}$$

Here, we provide a quantitative assessment of the RCMC error using the parameters of an L-band satellite SAR system ($\lambda = 0.238$ m, $R_0 = 800\,000$ m, $V_r = 7557.8$ m/s, azimuth bandwidth $B = 2000$ Hz, and $K_r = 2940.74$ GHz/s). The error attains its maximum value at the edge of the synthetic aperture, which corresponds to an azimuth frequency of approximately $f_\eta = B/2$. Even with a very large f_{ion} of 1 Hz, the error at this position is 0.20 m, which is 0.20% of the original RCM magnitude and approximately 1/10 of a range resolution cell. Therefore, its effects can be neglected. As for the higher order ionospheric phase components, their effects are even smaller, as their magnitudes are typically smaller than that of the linear ionospheric phase component.

Regarding SRC, it should be noted that the squint angles of most current spaceborne SAR systems are minimal. For example, ALOS-2 has a squint angle of less than 0.05° . Under

such small-squint-angle conditions, SRC can generally be neglected. Nevertheless, we demonstrate that the ionospheric effect on SRC is negligible.

The SAR signal without ionospheric effects in the range–Doppler domain is given by [26]

$$\begin{aligned} S_0(\tau, f_\eta) &= A'_0 w_r \left[\tau - \frac{2R_0}{cD(f_\eta, V_r)} \right] W_a(f_\eta - f_{\eta_c}) \\ &\quad \times \exp \left\{ -j \frac{4\pi R_0 D(f_\eta, V_r) f_0}{c} \right\} \\ &\quad \times \exp \left\{ j\pi K_m \left[\tau - \frac{2R_0}{cD(f_\eta, V_r)} \right]^2 \right\} \quad (82) \end{aligned}$$

where A'_0 is a complex constant,

$$D(f_\eta, V_r) = \sqrt{1 - \frac{c^2 f_\eta^2}{4V_r^2 f_0^2}} \quad (83)$$

is the migration factor, and

$$K_m = \frac{K_r}{1 - cR_0 f_\eta^2 K_r / [2V_r^3 f_0^3 D^3(f_\eta, V_r)]} \quad (84)$$

is the modified range chirp FM rate. Other parameters are identical to those in (1) and (79). The range–azimuth coupling-induced range FM rate deviation, $K_m - K_r$, necessitates SRC.

We again introduce the linear ionospheric phase component used in the analysis of RCMC. The signal then becomes

$$\begin{aligned} S_{\text{ion}}(\tau, f_\eta) &= A'_0 w_r \left[\tau - \frac{2R_0}{cD(f_\eta - f_{\text{ion}}, V_r)} \right] W_a(f_\eta - f_{\text{ion}} - f_{\eta_c}) \\ &\quad \times \exp \left\{ j2\pi f_{\text{ion}} \frac{f_{\eta_c}}{K_a} \right\} \\ &\quad \times \exp \left\{ -j \frac{4\pi R_0 D(f_\eta - f_{\text{ion}}, V_r) f_0}{c} \right\} \\ &\quad \times \exp \left\{ j\pi K_{m,\text{ion}} \left[\tau - \frac{2R_0}{cD(f_\eta - f_{\text{ion}}, V_r)} \right]^2 \right\} \quad (85) \end{aligned}$$

where

$$D(f_\eta - f_{\text{ion}}, V_r) = \sqrt{1 - \frac{c^2 (f_\eta - f_{\text{ion}})^2}{4V_r^2 f_0^2}} \quad (86)$$

and

$$\begin{aligned} K_{m,\text{ion}} &= \frac{K_r}{1 - cR_0 (f_\eta - f_{\text{ion}})^2 K_r / [2V_r^3 f_0^3 D^3(f_\eta - f_{\text{ion}}, V_r)]}. \quad (87) \end{aligned}$$

The variation in K_m is $K_{m,\text{ion}} - K_m$.

We also use the L-band satellite SAR system parameters to provide a quantitative assessment of the resulting error. At the edge of the synthetic aperture, which corresponds to an azimuth frequency of approximately $f_\eta = B/2$, $K_m - K_r$ reaches its maximum of 1 202 370.94 Hz/s. Here, $K_{m,\text{ion}} - K_m$

is -2404.43 Hz/s, representing only 0.20% of $K_m - K_r$. Therefore, its effects can be neglected. As for the higher order ionospheric phase components, their effects are again even smaller.

To summarize, the range–azimuth coupling effects primarily occur during RCMC and SRC, but these effects are negligible. Accounting for them does not significantly improve the precision and accuracy of the derived effects originating from the along-track ionospheric variations within the synthetic aperture; instead, it would complicate the theoretical analysis and make it challenging to obtain closed-form results.

APPENDIX B FOCUSING OF AZIMUTH SIGNAL

The azimuth matched filter can be derived in either the time domain or the frequency domain. Here, we briefly demonstrate that the two filters yield equivalent results.

Assume that the SAR signal of a point target is

$$s(\eta) = \text{rect}\left(\frac{\eta - \eta_c}{T}\right) \exp\{j\pi K \eta^2\} \quad (88)$$

where η is the time, η_c is a constant time offset corresponding to the Doppler centroid time of the azimuth SAR signal, T is the pulse duration, and K is the linear FM rate. Its spectrum can be derived using the POSP

$$S(f) = \mathcal{F}\{s(\eta)\} = \text{rect}\left(\frac{f + f_c}{|K|T}\right) \exp\left\{-j\pi \frac{f^2}{K}\right\} \quad (89)$$

where $f_c = -K\eta_c$. Here, the constant term resulting from POSP is omitted. The frequency-domain matched filter is then designed to cancel the quadratic phase term in the spectrum

$$H(f) = \text{rect}\left(\frac{f + f_c}{|K|T}\right) \exp\left\{j\pi \frac{f^2}{K}\right\}. \quad (90)$$

The output of the matched filter is

$$\begin{aligned} s_{\text{out}}(\eta) &= \mathcal{F}^{-1}\{S(f)H(f)\} \\ &= \mathcal{F}^{-1}\left\{\text{rect}\left(\frac{f + f_c}{|K|T}\right)\right\} \\ &= \text{sinc}(KT) \exp\{-j2\pi f_c \eta\}. \end{aligned} \quad (91)$$

The peak position of the focused target is located at $\eta = 0$, which corresponds to the zero Doppler time of the azimuth SAR signal.

Since the actual azimuth signal is not exactly a linear FM signal, as assumed in (88), it is difficult to derive its exact analytical matched filter. In practice, a more common approach used in azimuth focusing programs is to compute the matched filter in the time domain. The time-domain matched filter is the time-reversed, complex conjugate of $s(\eta)$

$$h(\eta) = s^*(-\eta) = \text{rect}\left(\frac{\eta + \eta_c}{T}\right) \exp\{-j\pi K \eta^2\}. \quad (92)$$

The output of this matched filter is

$$\begin{aligned} s_{\text{out}}(\eta) &= \mathcal{F}^{-1}\{\mathcal{F}\{s(\eta)\}\mathcal{F}\{h(\eta)\}\} \\ &= \mathcal{F}^{-1}\left\{\text{rect}\left(\frac{f + f_c}{|K|T}\right)\right\} \\ &= \text{sinc}(KT) \exp\{-j2\pi f_c \eta\} \end{aligned} \quad (93)$$

leading to the same result as that obtained with the frequency-domain matched filter.

APPENDIX C SOLUTION OF $s_{\text{quadratic}}(\eta)$

We denote the quadratic ionospheric phase component coefficient as ΔK_a , e.g., $\Delta K_a = K_{\text{ion}}$ in (20). We approximate the envelope $W_a(\cdot)$ with a rectangular function to simplify the derivation. Then, $S_{\text{quadratic}}(f_\eta)$ can be given by

$$\begin{aligned} S_{\text{quadratic}}(f_\eta) &= W_a(f_\eta) \exp\left\{j\pi \frac{\Delta K_a}{K_a^2} f_\eta^2\right\} \\ &\approx \text{rect}\left(\frac{f_\eta}{B}\right) \exp\left\{j\pi \frac{\Delta K_a}{K_a^2} f_\eta^2\right\} \\ &\approx \text{rect}\left(\frac{f_\eta}{B}\right) \exp\{j\pi \mu f_\eta^2\} \end{aligned} \quad (94)$$

where $\mu = \Delta K_a / K_a^2$. The time-domain signal is then

$$\begin{aligned} s_{\text{quadratic}}(\eta) &= \int_{-\infty}^{\infty} S_{\text{quadratic}}(f_\eta) \exp\{j2\pi f_\eta \eta\} df_\eta \\ &= \int_{f_L}^{f_H} \exp\{j\pi \mu f_\eta^2\} \exp\{j2\pi f_\eta \eta\} df_\eta \end{aligned} \quad (95)$$

where $f_H = B/2$, and $f_L = -B/2$. The amplitude of the above expression is symmetric about the zero point. Because direct evaluation of this integral is challenging, we use the Fresnel integrals to obtain a solution.

For the case of $\mu > 0$ ($\Delta K_a > 0$), $s_{\text{quadratic}}(\eta)$ can be written as

$$\begin{aligned} s_{\text{quadratic}}(\eta) &= \int_{f_L}^{f_H} \exp\left\{j\frac{\pi}{2} \left[2\mu \left(f_\eta + \frac{\eta}{\mu}\right)^2 - 2\mu \left(\frac{\eta}{\mu}\right)^2\right]\right\} df_\eta \\ &= \exp\left\{-j\pi \frac{\eta^2}{\mu}\right\} \int_{f_L}^{f_H} \exp\left\{j\frac{\pi}{2} 2\mu \left(f_\eta + \frac{\eta}{\mu}\right)^2\right\} df_\eta \\ &= \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi \frac{\eta^2}{\mu}\right\} \int_{g_L(\eta)}^{g_H(\eta)} \exp\left\{j\frac{\pi}{2} g^2\right\} dg \\ &= \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi \frac{\eta^2}{\mu}\right\} \\ &\quad \times \left(\int_0^{g_H(\eta)} \exp\left\{j\frac{\pi}{2} g^2\right\} dg - \int_0^{g_L(\eta)} \exp\left\{j\frac{\pi}{2} g^2\right\} dg\right) \end{aligned} \quad (96)$$

where $g = (2\mu)^{1/2}(f_\eta + \eta/\mu)$, $g_L(\eta) = (2\mu)^{1/2}(f_L + \eta/\mu) = (2\mu)^{1/2}(-B/2 + \eta/\mu)$, and $g_H(\eta) = (2\mu)^{1/2}(f_H + \eta/\mu) = (2\mu)^{1/2}(B/2 + \eta/\mu)$.

Conversely, when $\mu < 0$ ($\Delta K_a < 0$), $s_{\text{quadratic}}(\eta)$ becomes

$$\begin{aligned} s_{\text{quadratic}}(\eta) &= \frac{1}{\sqrt{2|\mu|}} \exp\left\{-j\pi \frac{\eta^2}{\mu}\right\} \\ &\quad \times \left(\int_0^{g_H(\eta)} \exp\left\{-j\frac{\pi}{2} g^2\right\} dg - \int_0^{g_L(\eta)} \exp\left\{-j\frac{\pi}{2} g^2\right\} dg\right) \end{aligned} \quad (97)$$

where $g = (2|\mu|)^{1/2}(f_\eta + \eta/\mu)$, $g_L(\eta) = (2|\mu|)^{1/2}(-B/2 + \eta/\mu)$,

and $g_H(\eta) = (2|\mu|)^{1/2}(B/2 + \eta/\mu)$.

By applying the Fresnel cosine integral and the Fresnel sine integral

$$\begin{cases} C(u) = \int_0^u \cos\left(\frac{\pi}{2}g^2\right) dg \\ S(u) = \int_0^u \sin\left(\frac{\pi}{2}g^2\right) dg \end{cases} \quad (98)$$

and Euler's formula

$$\exp\{j\theta\} = \cos(\theta) + j\sin(\theta) \quad (99)$$

the exponential functions in the integrand can be expressed in (100). Combining (96), (97), and (100), $s_{\text{quadratic}}(\eta)$ can be expressed in (101). Thus, it indicates that the quadratic ionospheric phase component introduces a phase error relative to the original SAR signal.

APPENDIX D

CLOSED-FORM APPROXIMATION OF $\Phi_{\text{quadratic}}(\eta)$

Using the power-series expansions of the Fresnel integrals, the phase of $s_{\text{quadratic}}(\eta)$, denoted by $\Phi_{\text{quadratic}}(\eta)$, can be approximated in a closed form.

Using Euler's formula, the complex Fresnel integrals in (96) and (97) can be expressed in terms of the Fresnel cosine integral and the Fresnel sine integral, as shown in (100). The Fresnel cosine integral and the Fresnel sine integral can be expanded into their respective power-series forms as given in (102) [33], where

$$\begin{cases} C_{4k+1} = (-1)^k \frac{(\pi/2)^{2k}}{(2k)!(4k+1)} \\ S_{4k+3} = (-1)^k \frac{(\pi/2)^{2k+1}}{(2k+1)!(4k+3)}. \end{cases} \quad (103)$$

By retaining the first two terms of the power-series expansions, the complex Fresnel integrals, as shown in (100), can be approximated as

$$\begin{cases} \int_0^u \exp\left\{j\frac{\pi}{2}g^2\right\} dg \approx u + j\frac{\pi}{6}u^3 \\ \int_0^u \exp\left\{-j\frac{\pi}{2}g^2\right\} dg \approx u - j\frac{\pi}{6}u^3. \end{cases} \quad (104)$$

Following (96), the expression for $s_{\text{quadratic}}(\eta)$ for $\mu > 0$ can be written as

$$\begin{aligned} s_{\text{quadratic}}(\eta) &= \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ &\quad \times \left(\int_0^{g_H(\eta)} \exp\left\{j\frac{\pi}{2}g^2\right\} dg - \int_0^{g_L(\eta)} \exp\left\{j\frac{\pi}{2}g^2\right\} dg \right) \\ &\approx \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ &\quad \times \left[\left[g_H(\eta) + j\frac{\pi}{6}g_H^3(\eta) \right] - \left[g_L(\eta) + j\frac{\pi}{6}g_L^3(\eta) \right] \right] \\ &= \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ &\quad \times \left[\left[g_H(\eta) - g_L(\eta) \right] + j\frac{\pi}{6} \left[g_H^3(\eta) - g_L^3(\eta) \right] \right]. \end{aligned} \quad (105)$$

Following (97), when $\mu < 0$, the signal becomes

$$\begin{aligned} s_{\text{quadratic}}(\eta) &= \frac{1}{\sqrt{2|\mu|}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ &\quad \times \left(\int_0^{g_H(\eta)} \exp\left\{-j\frac{\pi}{2}g^2\right\} dg - \int_0^{g_L(\eta)} \exp\left\{-j\frac{\pi}{2}g^2\right\} dg \right) \end{aligned}$$

$$\begin{cases} \int_0^u \exp\left\{j\frac{\pi}{2}g^2\right\} dg = \int_0^u \cos\left(\frac{\pi}{2}g^2\right) dg + j \int_0^u \sin\left(\frac{\pi}{2}g^2\right) dg = C(u) + jS(u) \\ \int_0^u \exp\left\{-j\frac{\pi}{2}g^2\right\} dg = \int_0^u \cos\left(\frac{\pi}{2}g^2\right) dg - j \int_0^u \sin\left(\frac{\pi}{2}g^2\right) dg = C(u) - jS(u). \end{cases} \quad (100)$$

$$s_{\text{quadratic}}(\eta) = \begin{cases} \frac{1}{\sqrt{2\mu}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ \times \left\{ \left[C\left(\sqrt{2\mu}\left(\frac{B}{2} + \frac{\eta}{\mu}\right)\right) - C\left(\sqrt{2\mu}\left(-\frac{B}{2} + \frac{\eta}{\mu}\right)\right) \right] + j \left[S\left(\sqrt{2\mu}\left(\frac{B}{2} + \frac{\eta}{\mu}\right)\right) - S\left(\sqrt{2\mu}\left(-\frac{B}{2} + \frac{\eta}{\mu}\right)\right) \right] \right\}, \mu > 0 \\ \frac{1}{\sqrt{2|\mu|}} \exp\left\{-j\pi\frac{\eta^2}{\mu}\right\} \\ \times \left\{ \left[C\left(\sqrt{2|\mu|}\left(\frac{B}{2} + \frac{\eta}{\mu}\right)\right) - C\left(\sqrt{2|\mu|}\left(-\frac{B}{2} + \frac{\eta}{\mu}\right)\right) \right] - j \left[S\left(\sqrt{2|\mu|}\left(\frac{B}{2} + \frac{\eta}{\mu}\right)\right) - S\left(\sqrt{2|\mu|}\left(-\frac{B}{2} + \frac{\eta}{\mu}\right)\right) \right] \right\}, \mu < 0 \end{cases} \quad (101)$$

$$\begin{cases} \int_0^u \cos\left(\frac{\pi}{2}g^2\right) dg = \sum_{k=0}^{\infty} C_{4k+1} u^{4k+1} = u - \frac{\pi^2}{40}u^5 + \frac{\pi^4}{3456}u^9 + \dots \\ \int_0^u \sin\left(\frac{\pi}{2}g^2\right) dg = \sum_{k=0}^{\infty} S_{4k+3} u^{4k+3} = \frac{\pi}{6}u^3 - \frac{\pi^3}{336}u^7 + \frac{\pi^5}{42240}u^{11} + \dots \end{cases} \quad (102)$$

$$\begin{aligned}
&\approx \frac{1}{\sqrt{2|\mu|}} \exp \left\{ -j\pi \frac{\eta^2}{\mu} \right\} \\
&\quad \times \left\{ \left[g_H(\eta) - j\frac{\pi}{6} g_H^3(\eta) \right] - \left[g_L(\eta) - j\frac{\pi}{6} g_L^3(\eta) \right] \right\} \\
&= \frac{1}{\sqrt{2|\mu|}} \exp \left\{ -j\pi \frac{\eta^2}{\mu} \right\} \\
&\quad \times \left\{ \left[g_H(\eta) - g_L(\eta) \right] - j\frac{\pi}{6} \left[g_H^3(\eta) - g_L^3(\eta) \right] \right\}. \quad (106)
\end{aligned}$$

Thus, its phase is given by

$$\begin{aligned}
&\Phi_{\text{quadratic}}(\eta) \\
&\approx \begin{cases} -\pi \frac{\eta^2}{\mu} + \arctan \left(\frac{\pi g_H^3(\eta) - g_L^3(\eta)}{6 g_H(\eta) - g_L(\eta)} \right), \mu > 0 \\ -\pi \frac{\eta^2}{\mu} - \arctan \left(\frac{\pi g_H^3(\eta) - g_L^3(\eta)}{6 g_H(\eta) - g_L(\eta)} \right), \mu < 0 \end{cases} \\
&\approx \begin{cases} -\pi \frac{\eta^2}{\mu} + \frac{\pi g_H^3(\eta) - g_L^3(\eta)}{6 g_H(\eta) - g_L(\eta)}, \mu > 0 \\ -\pi \frac{\eta^2}{\mu} - \frac{\pi g_H^3(\eta) - g_L^3(\eta)}{6 g_H(\eta) - g_L(\eta)}, \mu < 0. \end{cases} \quad (107)
\end{aligned}$$

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REFERENCES

- [1] T. Tadono, M. Ohki, T. Motohka, A. Rosenqvist, and S. Sobue, "Advanced land observing satellite-2 'DAICHI-2' (ALOS-2): Overview and operational status," *IEEE Geosci. Remote Sens. Mag. (replaces Newsletter)*, vol. 13, no. 2, pp. 41–48, Jun. 2025, doi: [10.1109/MGRS.2025.3535720](#).
- [2] R. Wang et al., "LuTan-1: An innovative L-band spaceborne bistatic interferometric synthetic aperture radar mission," *IEEE Geosci. Remote Sens. Mag.*, vol. 13, no. 2, pp. 58–78, Jun. 2025, doi: [10.1109/MGRS.2024.3478761](#).
- [3] M. Ohki, T. Motohka, Y. Kankaku, S. H. Miura, T. Tadono, and Y. Arikawa, "Advanced land observing satellite-4: Mission concepts and technical overview: Pioneering a new era of L-band SAR," *IEEE Geosci. Remote Sens. Mag.*, vol. 13, no. 2, pp. 35–40, Jun. 2025, doi: [10.1109/MGRS.2025.3529037](#).
- [4] P. A. Rosen et al., "The NASA-ISRO SAR mission: A summary," *IEEE Geosci. Remote Sens. Mag.*, vol. 13, no. 2, pp. 8–34, Jun. 2025, doi: [10.1109/MGRS.2025.3578258](#).
- [5] F. De Zan, M. Zonno, and P. López-Dekker, "Phase inconsistencies and multiple scattering in SAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 53, no. 12, pp. 6608–6616, Dec. 2015, doi: [10.1109/TGRS.2015.2444431](#).
- [6] F. De Zan and G. Gamba, "Vegetation and soil moisture inversion from SAR closure phases: First experiments and results," *Remote Sens. Environ.*, vol. 217, pp. 562–572, Nov. 2018, doi: [10.1016/j.rse.2018.08.034](#).
- [7] E. Wig, R. Michaelides, and H. Zebker, "Fine-resolution measurement of soil moisture from cumulative InSAR closure phase," *IEEE Trans. Geosci. Remote Sens.*, vol. 62, 2024, Art. no. 5212315, doi: [10.1109/TGRS.2024.3399069](#).
- [8] Y. Zheng, H. Fattahi, P. Agram, M. Simons, and P. Rosen, "On closure phase and systematic bias in multilooked SAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5226611, doi: [10.1109/TGRS.2022.3167648](#).
- [9] H. Ansari, F. De Zan, and A. Parizzi, "Study of systematic bias in measuring surface deformation with SAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 59, no. 2, pp. 1285–1301, Feb. 2021, doi: [10.1109/TGRS.2020.3003421](#).
- [10] C. Liang, E. J. Fielding, Z. Liu, T. Motohka, R. Natsuaki, and S.-H. Yun, "An analysis of the potentials of L-band SAR satellites for measuring azimuth motion," *Remote Sens. Environ.*, vol. 315, Dec. 2024, Art. no. 114426.
- [11] G. Gomba, A. Parizzi, F. De Zan, M. Eineder, and R. Bamler, "Toward operational compensation of ionospheric effects in SAR interferograms: The split-spectrum method," *IEEE Trans. Geosci. Remote Sens.*, vol. 54, no. 3, pp. 1446–1461, Mar. 2016, doi: [10.1109/TGRS.2015.2481079](#).
- [12] P. A. Rosen, S. Hensley, and C. Chen, "Measurement and mitigation of the ionosphere in L-band interferometric SAR data," in *Proc. IEEE Radar Conf.*, Arlington, VA, USA, May 2010, pp. 1459–1463, doi: [10.1109/RADAR.2010.5494385](#).
- [13] R. Brcic, A. Parizzi, M. Eineder, R. Bamler, and F. Meyer, "Estimation and compensation of ionospheric delay for SAR interferometry," in *Proc. IEEE Int. Geosci. Remote Sens. Symp.*, Honolulu, HI, USA, Jul. 2010, pp. 2908–2911, doi: [10.1109/IGARSS.2010.5652231](#).
- [14] A. L. Gray, K. E. Mattar, and G. Sofko, "Influence of ionospheric electron density fluctuations on satellite radar interferometry," *Geophys. Res. Lett.*, vol. 27, no. 10, pp. 1451–1454, May 2000, doi: [10.1029/2000gl000016](#).
- [15] U. Wegmuller, C. Werner, T. Strozzi, and A. Wiesmann, "Ionospheric electron concentration effects on SAR and INSAR," in *Proc. IEEE Int. Symp. Geosci. Remote Sens.*, Denver, CO, USA, Jul. 2006, pp. 3731–3734.
- [16] C. Liang and E. J. Fielding, "Measuring azimuth deformation with L-band ALOS-2 ScanSAR interferometry," *IEEE Trans. Geosci. Remote Sens.*, vol. 55, no. 5, pp. 2725–2738, May 2017, doi: [10.1109/TGRS.2017.2653186](#).
- [17] C. Liang, Z. Liu, E. J. Fielding, and R. Bürgmann, "InSAR time series analysis of L-band wide-swath SAR data acquired by ALOS-2," *IEEE Trans. Geosci. Remote Sens.*, vol. 56, no. 8, pp. 4492–4506, Aug. 2018, doi: [10.1109/TGRS.2018.2821150](#).
- [18] A. C. Chen and H. A. Zebker, "Reducing ionospheric effects in InSAR data using accurate coregistration," *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 1, pp. 60–70, Jan. 2014, doi: [10.1109/TGRS.2012.2236098](#).
- [19] J. Chen and H. A. Zebker, "Ionospheric artifacts in simultaneous L-band InSAR and GPS observations," *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 4, pp. 1227–1239, Apr. 2012, doi: [10.1109/TGRS.2011.2164805](#).
- [20] W. Mao, X. Wang, G. Liu, R. Zhang, Y. Shi, and S. Pirasteh, "Estimation and compensation of ionospheric phase delay for multi-aperture InSAR: An azimuth split-spectrum interferometry approach," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5209414, doi: [10.1109/TGRS.2021.3095272](#).
- [21] T. Yamashita, Y. Morishita, and T. Kobayashi, "Mitigation of ionospheric noise in azimuth offset based on the split-spectrum method," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5206411, doi: [10.1109/TGRS.2021.3073511](#).
- [22] Q. Wang, J. Hu, A. Guo, and R. Gui, "Ionospheric delay phase estimation and correction for multiple-aperture InSAR: Azimuth group phase delay method," *IEEE Trans. Geosci. Remote Sens.*, vol. 62, 2024, Art. no. 5222909, doi: [10.1109/TGRS.2024.3417906](#).
- [23] F. Meyer, R. Bamler, N. Jakowski, and T. Fritz, "The potential of low-frequency SAR systems for mapping ionospheric TEC distributions," *IEEE Geosci. Remote Sens. Lett.*, vol. 3, no. 4, pp. 560–564, Oct. 2006, doi: [10.1109/LGRS.2006.882148](#).
- [24] S. Quegan and J. Lamont, "Ionospheric and tropospheric effects on synthetic aperture radar performance," *Int. J. Remote Sens.*, vol. 7, no. 4, pp. 525–539, Apr. 1986.
- [25] D. P. Belcher, "Theoretical limits on SAR imposed by the ionosphere," *IET Radar, Sonar Navigat.*, vol. 2, no. 6, pp. 435–448, Dec. 2008.
- [26] I. G. Cumming and F. H. Wong, *Digital Processing of Synthetic Aperture Radar Data: Algorithms and Implementation*. Norwood, MA, USA: Artech House, 2005.
- [27] J. C. Curlander and R. N. McDonough, *Synthetic Aperture Radar: Systems and Signal Processing*. New York, NY, USA: Wiley, 1991.
- [28] D. W. Lozier and F. W. J. Olver, "Numerical evaluation of special functions," in *Mathematics of Computation, 1943–1993: A Half Century of Computational Mathematics*. Providence, RI, USA: AMS, 1994, pp. 79–125.
- [29] R. F. Boisvert, "The architecture of an intelligent virtual mathematical software repository system," *Math. Comput. Simul.*, vol. 36, pp. 269–279, Oct. 1994.
- [30] R. Scheiber and A. Moreira, "Coregistration of interferometric SAR images using spectral diversity," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 5, pp. 2179–2191, Sep. 2000, doi: [10.1109/36.868876](#).

- [31] N. B. D. Bechor and H. A. Zebker, "Measuring two-dimensional movements using a single InSAR pair," *Geophys. Res. Lett.*, vol. 33, no. 16, Aug. 2006, Art. no. L16311, doi: [10.1029/2006gl026883](https://doi.org/10.1029/2006gl026883).
- [32] C. Oliver and S. Quegan, "Principles of SAR image formation," in *Understanding Synthetic Aperture Radar Images*, vol. 6. Raleigh, NC, USA: SciTech, 2004, ch. 2, sec. 2, pp. 11–42.
- [33] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. Washington, DC, USA: U.S. Government Printing Office, 1972.



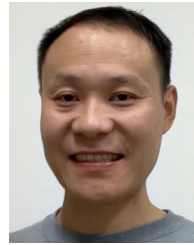
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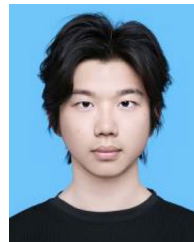


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